Free Fall

Held down by shoulder bars, you stop for a moment at the top. The supports under the car are released, and you're in free fall, dropping faster and faster for a time interval of 1.5 s. How far will you drop and how fast will you be going the instant before the car reaches the bottom?

5

A Mathematical Model of Motion

F or pure heart-stopping excitement, nothing beats a ride on the Demon Drop at Cedar Point Amusement Park. Enthusiasts say its free-fall drop of 60 feet is the ultimate rush. Isn't the excitement of most of the rides the rapid and unexpected changes in speed and direction? And what about those roller-coaster lurches and sudden stops; the swaying and the hairpin turns? Your favorite rides are probably the ones with the sharpest turns and the most precipitous drops. Your ticket to ride entitles you to scream while your stomach sinks into your shoes.

What's hard to imagine is that the Demon Drop and its neighboring roller coaster, the Millennium 2000, depend on the same basic principles of physics. Once your Demon Drop car reaches the top of the tower, no motors, engines, pulleys, or other energy source interferes with your fall. The same is true for roller coasters. Initially, cables pull the coaster cars to the top of the first hill. From there the laws of physics take charge, propelling the rides downhill, up again, through loops and spirals at speeds of 37 km/h (60 mph) or more. In fact, many amusement park rides, from the gently circling carousel and Ferris wheel to the tilt-o-whirl and roller coaster, offer rides that can be specified and described in mathematical terms.

The vectors you learned about in Chapter 4, together with the concepts of velocity and acceleration, can help explain why amusement parks are fun. They also explain how such hair-raising rides as the Demon Drop and Millennium 2000 roller coasters simulate danger while remaining safe.

WHAT YOU'LL LEARN

- You will continue your study of average and instantaneous velocity, and acceleration.
- You will use graphs and equations to solve problems involving moving objects, including freely falling objects.

WHY IT'S IMPORTANT

- The rapid pace of life today means that modern cars, planes, elevators, and other people-moving vehicles often are designed to accelerate quickly from rest to the highest speed deemed safe, and then continue at that speed until they reach their destinations.

PHYSICS
Online

To find out more about motion, visit the Glencoe Science Web site at **science.glencoe.com**

CLICK HERE

5.1 Graphing Motion in One Dimension

OBJECTIVES

- **Interpret** graphs of position versus time for a moving object to determine the velocity of the object.

- **Describe** in words the information presented in graphs and **draw** graphs from descriptions of motion.

- **Write** equations that describe the position of an object moving at constant velocity.

You have learned how to describe motion in terms of words, sketches, and motion diagrams. In this chapter, you'll learn to represent one-dimensional motion by means of a graph of position versus time. Such a graph presents information not only about the displacement of an object, but also about its velocity.
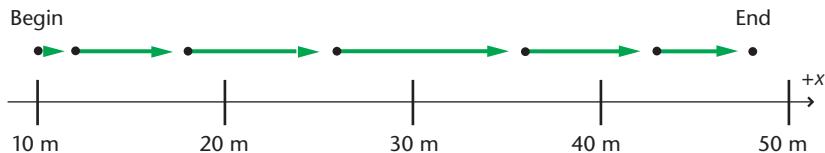
The other tool that is useful for certain kinds of motion is an equation that describes an object's displacement versus time. You can use a position-time graph and this equation to analyze the motion of an object mathematically and to make predictions about its position, velocity, and acceleration.
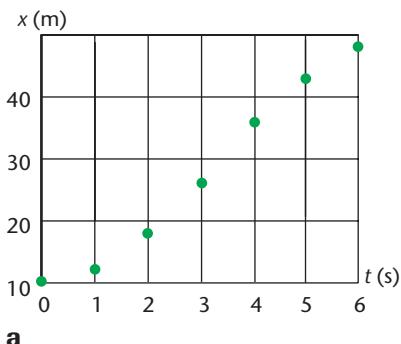
Position-Time Graphs

How could you make a graph of the position of the Demon Drop at various times? Such a graph would be a position-time graph. You will learn to make a *p-t* graph for the Demon Drop, but first, consider a simpler example. A physics student uses a camcorder to record the motion of a running back as he runs straight down the football field to make a touchdown. She records one frame each second and produces the motion diagram shown in **Figure 5–1.** From the motion diagram, the physics student obtains the data in **Table 5–1.** Because she chooses the *x*-coordinate axis, the symbol *x* in this problem represents distance from his own goal line.
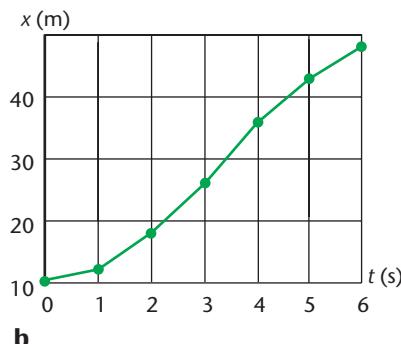
Notice the origin of the coordinate system in **Figure 5–1.** Time was set to zero when the running back began to move with the ball, but the origin of the *x*-axis was not chosen to be his initial position. Instead, he began 10 m from the origin.

Two graphs of the running back's motion are shown in **Figure 5–2.** In the first graph, only the recorded positions are shown. In the second graph, a curve connects each of the recorded points. These lines represent our best guess as to where the running back was in between the recorded points. You can see that this graph is not a picture of the path taken by the ball carrier as he was running; the graph is curved, but the path that he took down the field was not.

| TABLE 5–1	
Position versus Time	
Time, t (s)	Position, x (m)
0.0	10
1.0	12
2.0	18
3.0	26
4.0	36
5.0	43
6.0	48

FIGURE 5–1 The football player began at 10 m and ran in the positive *x* direction.

FIGURE 5–2 Only the plotted points are known. The lines joining the points are best guesses as to the position of the running back.

What is an instant of time? How long did the running back spend at any location? Each position has been linked to a time, but how long did that time last? You could say "an instant," but how long is that? If an instant lasts for any finite amount of time, then, because the running back would be at the same position during that time, he would be at rest. But a moving object cannot be at rest; thus, an instant is not a finite period of time. This means that an instant of time lasts zero seconds. The symbol x represents the instantaneous position of the running back at a particular instant of time.

Using a Graph to Find Out *Where* and *When*

When did the running back reach the 30-m mark? Where was he 4.5 s after he started running? These questions can be answered easily with a position-time graph, as you will see in the following example problem. Note that the questions are first restated in the language of physics in terms of positions and times.

Example Problem

Data from a Position–Time Graph

When did the running back reach the 30-m mark? Where was he after 4.5 seconds?

**Strategy:**

Restate the questions:
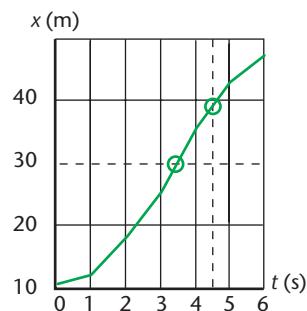
Question 1: At what time was the position of the object equal to 30 m?

Question 2: What was the position of the object at 4.5 s?

To answer question 1, examine the graph to find the intersection of the curve with a horizontal line at the 30-m mark.

To answer question 2, find the intersection of the curve with a vertical line at 4.5 s (halfway between 4 s and 5 s on this graph).
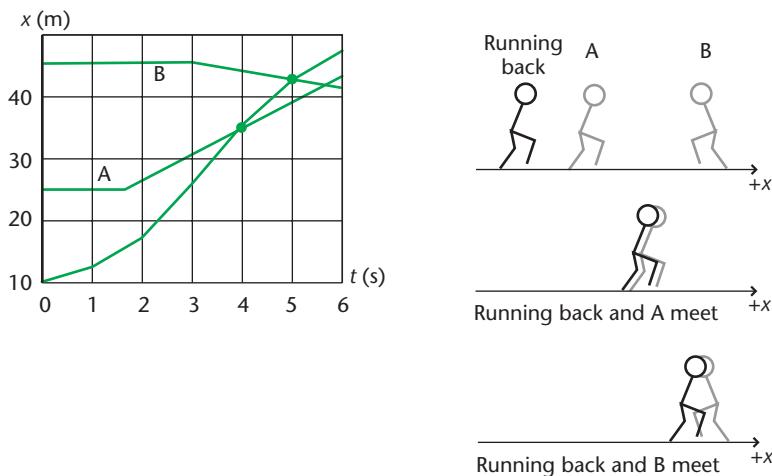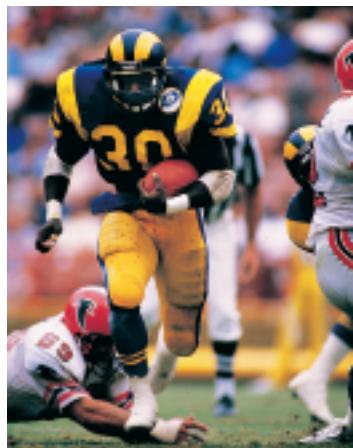
The two intersections are shown on the graph.

Graphing the Motion of Two or More Objects

Pictorial and graphical representations of the running back and two players, A and B, on the opposing team are shown in **Figure 5–3.** When and where would each of them have a chance to tackle the running back? First, you need to restate this question in physics terms: At what time do two objects have the same position? On a position-time graph, when do the curves representing the two objects intersect?

FIGURE 5–3 When and where can A and B tackle the running back?

Running back and A meet

Running back and B meet

Example Problem

Interpreting Position–Time Graphs

Where and when do defenders A and B have a chance to tackle the running back?

Strategy:

In **Figure 5–3,** the intersections of the curves representing the motion of defenders A and B with the curve representing the motion of the running back are points.

Defender A intersects with running back in 4 s at about 35 m.

Defender B intersects in 5 s at about 42 m.

From Graphs to Words and Back Again

To interpret a position-time graph in words, start by finding the position of the object at $t = 0$. You have already seen that the position of the object is not always zero when $t = 0$. Then, examine the curve to see whether the position increases, remains the same, or decreases with increasing time. Motion away from the origin in a positive direction has a positive velocity, and motion in the negative direction has a negative velocity. If there is no change in position, then the velocity is zero.

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Describing Motion from a Position-Time Graph

Describe the motion of the players in **Figure 5–3.**

Strategy:

The running back started at the 10-m mark and moved in the positive direction, that is, with a positive velocity.

Defender A started at 25 m. After waiting about 1.5 s, he also moved with positive velocity.

Defender B started at 45 m. After 3 s, he started running in the opposite direction, that is, with a negative velocity.

Practice Problems

1. Describe in words the motion of the four walkers shown by the four lines in **Figure 5–4.** Assume the positive direction is east and the origin is the corner of High Street.
2. Describe the motion of the car shown in **Figure 5–5.**
3. Answer the following questions about the car whose motion is graphed in **Figure 5–5.**
   a. When was the car 20 m west of the origin?
   b. Where was the car at 50 s?
   c. The car suddenly reversed direction. When and where did that occur?
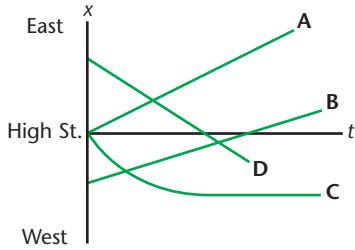

FIGURE 5–4
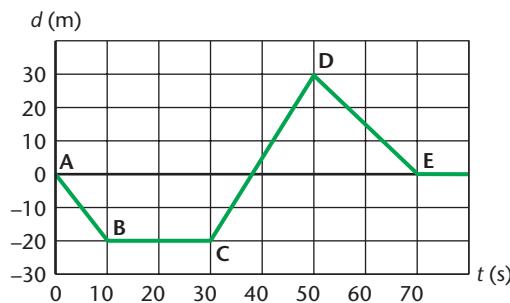
FIGURE 5–5

## Uniform Motion

If an airplane travels 75 m in a straight line in the first second of its flight, 75 m in the next second, and continues in this way, then it is moving with uniform motion. **Uniform motion** means that equal displacements occur during successive equal time intervals. A motion diagram and a position-time graph can be used to describe the uniform motion of the plane.

CONTENTS
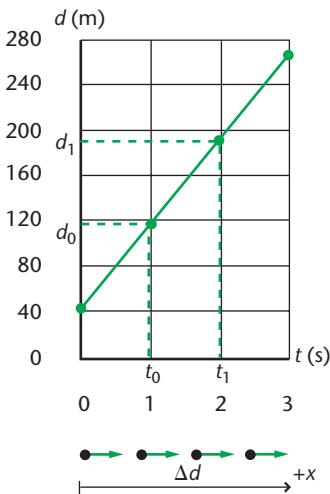
FIGURE 5–6 The regular change in position shows that this airplane is moving with uniform velocity.

FIGURE 5–7 You can tell in the graph below that the riders are traveling at different velocities because the three lines representing their velocities have different slopes.

The line drawn through the points representing the position of the plane each second is a straight line. Recall from Chapter 2 that one of the properties of a straight line is its slope. To find the slope, take the ratio of the vertical difference between two points on the line, the rise, to the horizontal difference between the same points, the run.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

Figure 5–6 shows how to determine the slope using the points at 1 s and 2 s. Any set of points would produce the same result because a straight line has a constant slope.

When the line is a position-time graph, then rise $= \Delta d$ and run $= \Delta t$, so the slope is the average velocity.

$$\text{slope} = \bar{v} = \frac{\Delta d}{\Delta t} = \frac{d_1 - d_0}{t_1 - t_0}$$
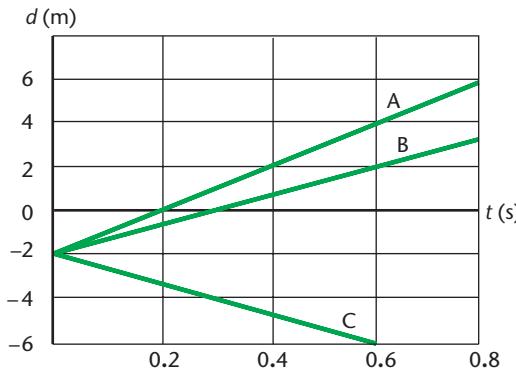
Thus, on a position-time graph, the slope of a straight line passing through the points on the graph at times t_0 and t_1 is the average velocity between any two times. In **Figure 5–6,** notice that when t_0 is 1 s and t_1 is 2 s, d_0 equals 115 m and d_1 equals 190 m. The average velocity of the airplane $\bar{v} = (190 \text{ m} - 115 \text{ m})/(2 \text{ s} - 1 \text{ s}) = 75$ m/s.

Note that the average velocity is not d/t, as you can see by calculating that ratio from the coordinates of the points on the graph. Check this for yourself using **Figure 5–6.** You will find that when $t = 2$ s and $d = 190$ m, the ratio d/t is 95 m/s, not 75 m/s, as calculated in the previous equation.

What do position-time graphs look like for different velocities? Look at the graph of three bike riders in **Figure 5–7.** Note the initial position of the riders. Rider A has a displacement of 4.0 m in 0.4 s, so the average velocity of rider A is the following.

$$\bar{v}_A = \frac{\Delta d}{\Delta t} = \frac{2.0 \text{ m} - (-2.0 \text{ m})}{0.4 \text{ s} - 0 \text{ s}} = \frac{4.0 \text{ m}}{0.4 \text{ s}} = 10 \text{ m/s}$$

What is the average velocity, $\bar{v}_B$, of rider B? The slope of the line is less than that for rider A, so the magnitude of the velocity of rider B should be smaller. With a displacement of 4.0 m in 0.6 s, the speed is 6.7 m/s, less than that of A, as expected.

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What about rider C? Although rider C moves 4.0 m in 0.6 s, her displacement, which is the final position minus the initial position, is -4.0 m. Thus, her average velocity is $\bar{v}_C = -6.7$ m/s. This tells you that rider C is moving in the negative direction. A line that slants downward and has a negative slope represents a negative average velocity. Recall that a velocity is negative if the direction of motion is opposite the direction you chose to be positive.

Practice Problems

4. For each of the position-time graphs shown in **Figure 5–8,**
   a. write a description of the motion.
   b. draw a motion diagram.
   c. rank the average velocities from largest to smallest.
5. Draw a position-time graph for a person who starts on the positive side of the origin and walks with uniform motion toward the origin. Repeat for a person who starts on the negative side of the origin and walks toward the origin.
6. Chris claims that as long as average velocity is constant, you can write $v = d/t$. Use data from the graph of the airplane's motion in **Figure 5–6** to convince Chris that this is not true.
7. Use the factor-label method to convert the units of the following average velocities.
   a. speed of a sprinter: 10.0 m/s into mph and km/h
   b. speed of a car: 65 mph into km/h and m/s
   c. speed of a walker: 4 mph into km/h and m/s
8. Draw a position-time graph of a person who walks one block at a moderate speed, waits a short time for a traffic light, walks the next block slowly, and then walks the final block quickly. All blocks are of equal length.
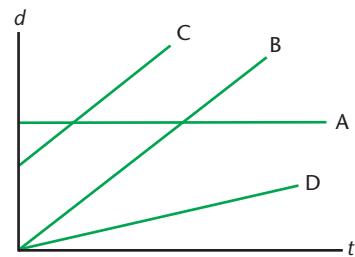

FIGURE 5–8

Using an Equation to Find Out *Where* and *When*

Uniform motion can be represented by an algebraic equation. Recall from Chapter 3 that the average velocity was defined in this way.

$$\textbf{Average Velocity} \quad \bar{v} = \frac{\Delta d}{\Delta t} = \frac{d_1 - d_0}{t_1 - t_0}$$

Note the absence of bold-face type. All algebra is done using the components of vectors and not the vectors themselves. Assume that you've chosen the origin of the time axis to be zero, so that $t_0 = 0$. Then, t_0 can be eliminated and the equation rearranged.

$$d_1 = d_0 + \bar{v}t_1$$

Pocket Lab

Uniform or Not?

Set up a U-channel on a book so that it acts as an inclined ramp, or make a channel from two metersticks taped together along the edges. Release a steel ball so that it rolls down the ramp. Using a stopwatch, measure the time it takes the ball to roll 0.40 m.

Analyze and Conclude Write a brief description of the motion of the ball. Predict how much time it would take the ball to roll 0.80 m. Explain your prediction.

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The equation can be made more general by letting t be any value of t_1 and d be the value of the position at that time. In addition, the distinction between velocity and average velocity is not needed because you will be working with the special case of constant velocity. This means that the average velocity between any two times will be the same as the constant instantaneous velocity, $\bar{v} = v$. The symbol v represents velocity, and d_0 represents the position at $t = 0$. The following equation is then obtained for the position of an object moving at constant velocity.

Position with Constant Velocity $\quad d = d_0 + vt$

The equation involves four quantities: the initial position, d_0, the constant velocity, v, the time, t, and the position at that time, d. If you are given three of these quantities, you can use the equation to find the fourth. When a problem is stated in words, you have to read it carefully to find out which three are given and which is unknown. When problems are given in graphical form, the slope of the curve tells you the constant velocity, and the point where the curve crosses the $t = 0$ line is the initial position. The following example problem illustrates the use of both a graph and the equation in solving a problem.

Example Problem

Finding Position from a Graph and an Equation

Write the equation that describes the motion of the airplane graphed in **Figure 5–6,** and find the position of the airplane at 2.5 s.

Calculate Your Answer

Known:

$v = 75$ m/s

$d_0 = 40$ m

$t = 2.5$ s

Unknown:

$d = ?$

Strategy:

The constant velocity is 75 m/s.

The curve intersects the $t = 0$ line at 40 m, so the initial position is 40 m.

You know the time, so the equation to use is $d = d_0 + vt$.

Calculations:

$d = d_0 + vt$

$d = 40$ m $+ (75$ m/s$)(2.5$ s$)$

$\quad = 230$ m

Check Your Answer

- Is the unit correct? m/s × s results in m.
- Does the sign make sense? It is positive, as it should be.
- Is the magnitude realistic? The result agrees with the value shown on the graph.

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9. Consider the motion of bike rider A in **Figure 5–7.**
   a. Write the equation that represents her motion.
   b. Where will rider A be at 1.0 s?
10. Consider the motion of bike rider C in **Figure 5–7.**
   a. Write the equation that represents her motion.
   b. When will rider C be at -10.0 m?
11. A car starts 2.0×10^2 m west of the town square and moves with a constant velocity of 15 m/s toward the east. Choose a coordinate system in which the x-axis points east and the origin is at the town square.
   a. Write the equation that represents the motion of the car.
   b. Where will the car be 10.0 min later?
   c. When will the car reach the town square?
12. At the same time the car in problem 11 left, a truck was 4.0×10^2 m east of the town square moving west at a constant velocity of 12 m/s. Use the same coordinate system as you did for problem 11.
   a. Draw a graph showing the motion of both the car and the truck.
   b. Find the time and place where the car passed the truck using both the graph and your two equations.

EARTH SCIENCE CONNECTION

Space Probes Scientists use other planets' gravity to alter the path of a space probe and increase its speed. The probe is programmed to pass near a planet. The planet's gravity bends the probe's trajectory and propels it away. In 1974, *Mariner 10* swung by Venus to gain energy for three passes by Mercury. In 1992, Jupiter's gravity allowed *Ulysses* to make a hairpin turn on its way to take photos of the sun's south pole.

5.1 Section Review

1. You drive at constant speed toward the grocery store, but halfway there you realize you forgot your list. You quickly turn around and return home at the same speed. Describe in words the position-time graph that would represent your trip.

2. A car drives 3.0 km at a constant speed of 45 km/h. You use a coordinate system with its origin at the point where the car started and the direction of the car as the positive direction. Your friend uses a coordinate system with its origin at the point where the car stopped and the opposite direction as the positive direction. Would the two of you agree on the car's position? Displacement? Distance? Velocity? Speed?

3. Write equations for the motion of the car just described in both coordinate systems.

4. **Critical Thinking** A police officer clocked a driver going 20 mph over the speed limit just as the driver passed a slower car. He arrested both drivers. The judge agreed that both were guilty, saying, "If the two cars were next to each other, they must have been going the same speed." Are the judge and police officer correct? Explain with a sketch, a motion diagram, and a position-time graph.

Graphing Velocity in One Dimension

OBJECTIVES

- **Determine,** from a graph of velocity versus time, the velocity of an object at a specified time.

- **Interpret** a *v-t* graph to find the time at which an object has a specific velocity.

- **Calculate** the displacement of an object from the area under a *v-t* curve.

You've learned how to draw a position-time graph for an object moving at a constant velocity, and how to use that graph to write an equation to determine the velocity of the object. You also have learned how to use both the graph and the equation to find an object's position at a specified time, as well as the time it takes the object to reach a specific position. Now you will explore the relationship between velocity and time when velocity is not constant.

Determining Instantaneous Velocity

What does a position-time graph look like when an object is going faster and faster? **Figure 5–9a** shows a different position-time record of an airplane flight. The displacements for equal time intervals in both the motion diagram and the graph get larger and larger. This means that the average velocity during each time interval, $\bar{v} = \Delta d/\Delta t$, also gets larger and larger. The motion is certainly not uniform. The instantaneous velocity cannot equal the average velocity.

How fast was the plane going at 1.5 s? The average velocity is the slope of the straight line connecting any two points. Using **Figure 5–9a,** you can see that the average velocity between 1 s and 2 s is $\Delta d/\Delta t$, that is $(10 \text{ m} - 4 \text{ m})/(1 \text{ s})$, or 6 m/s. But the velocity could have changed within that second. To be precise, more data are needed. **Figure 5–9b** shows the slope of the line connecting 1.25 s and 1.75 s. Because this time interval is half of the 1-s time interval used previously, the average velocity is probably closer to the instantaneous velocity at 1.5 s. You could continue the process of reducing the time interval and finding the ratio of the displacement to the time interval. Each time you reduce the time interval, the ratio is likely to be closer to the quantity called the
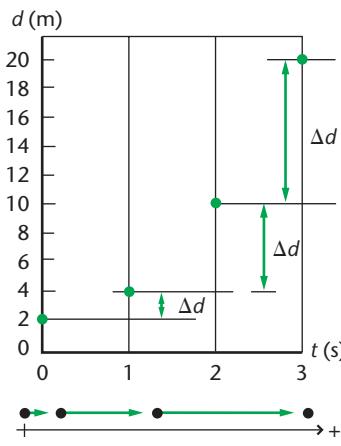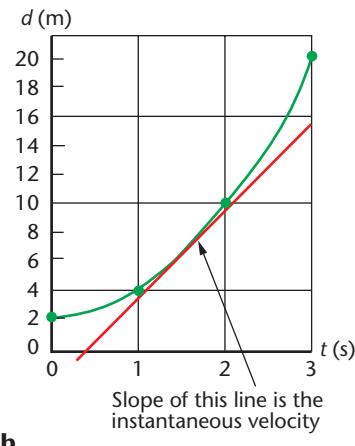
FIGURE 5–9 The displacement during equal time intervals increases as velocity increases. The graph in **b** shows that the slope of the tangent to the curve at any point is the instantaneous velocity at that time.

Slope of this line is the instantaneous velocity

a

b

instantaneous velocity. Finally, you would find that the slope of the line tangent to the position-time curve at a specific time is the instantaneous velocity, v, at that time.
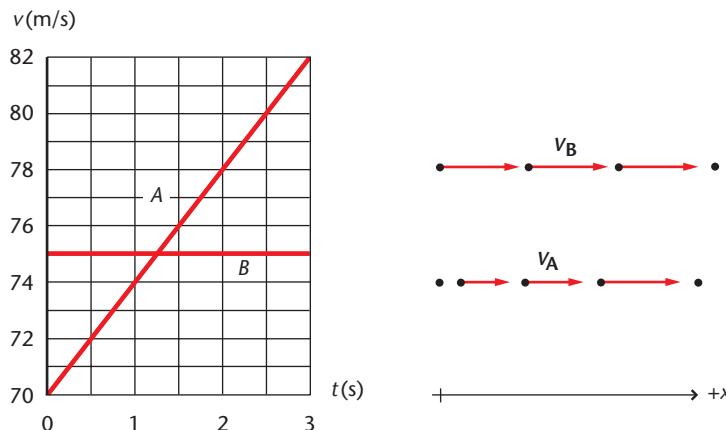
Velocity-Time Graphs

Now that you know what the velocity at an instant of time is and how to find it, you will be able to draw a graph of the velocity versus time of the airplane whose motion is not constant. Just as average velocity arrows on a motion diagram are drawn in red, the curves on a velocity-time (v-t) graph are also shown in red.

Motion diagrams and a velocity-time graph for the two airplanes, one with constant velocity and the other with increasing velocity, are shown in **Figure 5–10** for a 3-s time interval during their flights. Note that the velocities are given in m/s. The velocity of one airplane is increasing, while the velocity of the other airplane is constant. Which line on the graph represents the plane with uniform motion? Uniform motion, or constant velocity, is represented by a horizontal line on a v-t graph. Airplane B is traveling with a constant velocity of 75 m/s, or 270 km/h. The second line increases from 70 m/s to 82 m/s over a 3-s time interval. At 1.5 s, the velocity is 76 m/s. This is the slope of the tangent to the curve on a position-time graph at 1.5 s for the flight of airplane A.

What would the v-t graph look like if a plane were going at constant speed in the opposite direction? As long as you don't change the direction of the coordinate axis, the velocity would be negative, so the graph would be a horizontal line below the t-axis.

What can you learn from the intersection of the two lines on the graph? When the two v-t lines cross, the two airplanes have the same velocity. The planes do not necessarily have the same position, so they do not meet at this time. Velocity-time graphs give no information about position, although, as you will learn in the next section, you can use them to find displacement.

Math Handbook

To review **calculating the area under a graph,** see the Math Handbook, Appendix A, page 743.

FIGURE 5–10 The lines on the graph and the motion diagrams are two different ways of representing both constant and increasing velocity.

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v (m/s)
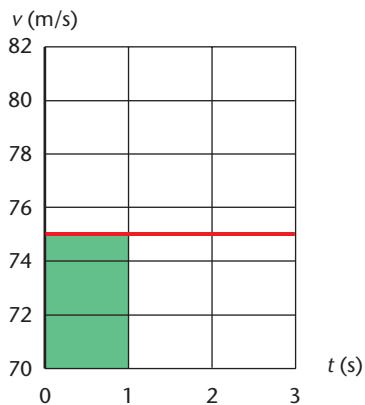
FIGURE 5–11 The displacement during a given time interval is the area under the curve of a v-t graph.

Displacement from a Velocity-Time Graph

For an object moving at constant velocity,

$$v = \bar{v} = \frac{\Delta d}{\Delta t}, \text{ so } \Delta d = v\Delta t.$$

As you can see in **Figure 5–11,** v is the height of the curve above the t-axis, while Δt is the width of the shaded rectangle. The area of the rectangle, then, is $v\Delta t$, or Δd. You can find the displacement of the object by determining the area under the v-t curve.

If the velocity is constant, the displacement is proportional to the width of the rectangle. Thus, if you plot the displacement versus time, you will get a straight line with slope equal to velocity. If the velocity is increasing, then the area of the rectangle increases in time, so the slope of a displacement-versus-time graph also increases.

Example Problem

Finding the Displacement of an Airplane from Its *v-t* Graph

Find the displacement of the plane in **Figure 5–11** that is moving at constant velocity after

a. 1.0 s.   **b.** 2.0 s.   **c.** 3.0 s.

Compare your results to the original position-time graph in **Figure 5–6.**

Calculate Your Answer

Known:

$v = 75$ m/s
$\Delta t = 1.0$ s, 2.0 s, and 3.0 s

Unknown:

Δd at 1.0 s, 2.0 s, and 3.0 s

Strategy:

The displacement is the area under the curve, or $\Delta d = v\Delta t$.

Calculations:

a. $\Delta d = v\Delta t = (75 \text{ m/s})(1.0 \text{ s}) = 75$ m
b. $\Delta d = v\Delta t = (75 \text{ m/s})(2.0 \text{ s}) = 150$ m
c. $\Delta d = v\Delta t = (75 \text{ m/s})(3.0 \text{ s}) = 225 \text{ m} = 230$ m

Check Your Answer

- Are the units correct? m/s $\times$ s = m.
- Do the signs make sense? They are all positive, as they should be.
- Are the magnitudes realistic? The calculated positions are different from those on the position-time graph: 115 m, 190 m, and 265 m. The differences occur because the initial position of the plane at $t = 0$ is 40 m, not zero. You must add the displacement of the airplane at $t = 0$ to the value calculated at each time.

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13. Use **Figure 5–10** to determine the velocity of the airplane that is speeding up at
    a. 1.0 s.
    b. 2.0 s.
    c. 2.5 s.
14. Use the factor-label method to convert the speed of the airplane whose motion is graphed in **Figure 5–6** (75 m/s) to km/h.
15. Sketch the velocity-time graphs for the three bike riders in **Figure 5–7.**
16. A car is driven at a constant velocity of 25 m/s for 10.0 min. The car runs out of gas, so the driver walks in the same direction at 1.5 m/s for 20.0 min to the nearest gas station. After spending 10.0 min filling a gasoline can, the driver walks back to the car at a slower speed of 1.2 m/s. The car is then driven home at 25 m/s (in the direction opposite that of the original trip).
    a. Draw a velocity-time graph for the driver, using seconds as your time unit. You will have to calculate the distance the driver walked to the gas station in order to find the time it took the driver to walk back to the car.
    b. Draw a position-time graph for the problem using the areas under the curve of the velocity-time graph.

Pocket Lab

Bowling Ball Displacement

Take a bowling ball and three stopwatches into the hallway. Divide into three groups. Have all timers start their watches when the ball is rolled. Group 1 should stop its watch when the ball has gone 10 m, group 2 should stop its watch when the ball has rolled 20 m, and group 3 should stop its watch when the ball has rolled 30 m.

Analyze and Conclude
Record the data and calculate the average speed for each distance. Could the average speed for 30 m be used to predict the time needed to roll 100 m? Why or why not?

5.2 Section Review

1. What information can you obtain from a velocity-time graph?
2. Two joggers run at a constant velocity of 7.5 m/s toward the east. At time $t = 0$, one is 15 m east of the origin; the other is 15 m west.
   a. What would be the difference(s) in the position-time graphs of their motion?
   b. What would be the difference(s) in their velocity-time graphs?
3. Explain how you would use a velocity-time graph to find the time at which an object had a specified velocity.
4. Sketch a velocity-time graph for a car that goes 25 m/s toward the east for 100 s, then 25 m/s toward the west for another 100 s.
5. **Critical Thinking** If the constant velocity on a *v-t* graph is negative, what is the sign of the area under the curve? Explain in terms of the displacement of the object whose motion is represented on the graph.

5.3 Acceleration

In Chapter 3, you learned how to use a motion diagram to get a feel for the average acceleration of an object. This method is illustrated in **Figure 5–12b** for the motion of two airplanes. Airplane A travels with non-uniform velocity, so the change in velocity, and thus the acceleration, is in the same direction as the velocity. Both velocity and acceleration have a positive sign. Airplane B travels with uniform velocity, so its acceleration is zero.

Determining Average Acceleration

Average acceleration is the rate of change of velocity between t_0 and t_1, and is represented by the following equation.
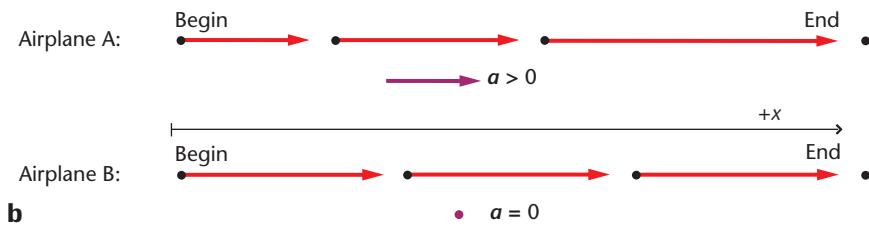
$$\textbf{Average Acceleration} \quad \bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_1 - v_0}{t_1 - t_0}$$

You can find this ratio by determining the slope of the velocity-time graph in the same way you found velocity from the slope of the position-time graph. For example, **Figure 5–12a** shows that in a 1-s time interval, the velocity of plane A increases by 4 m/s. That is, $\Delta v = 4$ m/s and $\Delta t = 1$ s, so $\bar{a} = \Delta v/\Delta t = 4$ m/s^2.

Constant and Instantaneous Acceleration

Recall that an object undergoes uniform motion, or constant velocity, if the slope of the position-time graph is constant. Does the slope of the velocity-time graph for the accelerating airplane of **Figure 5–12a** change? No, it rises by 4 m/s every second. This type of motion, which can be described by a constant slope on a velocity-time graph, is called **constant acceleration.**

What if the slope of a *v-t* graph isn't constant? You learned that you could find the instantaneous velocity by finding the slope of the tangent to the curve on the position-time graph. In the same way, you can find **instantaneous acceleration,** *a*, as the slope of the tangent to the curve on a velocity-time graph. Instantaneous acceleration is the acceleration of an object at an instant of time. Consider the velocity-time graph in the example problem.

FIGURE 5–12 Graphs and motion diagrams are useful in differentiating motion having uniform velocity and motion that is accelerated.

a

b

CONTENTS

Determining Velocity and Acceleration from a Graph

How would you describe the sprinter's velocity and acceleration as shown on the graph?
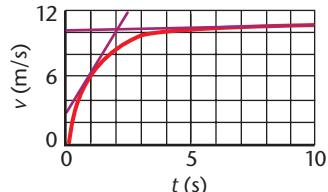
Calculate Your Answer

Strategy:

From the graph, note that the sprinter's velocity starts at zero, increases rapidly for the first few seconds, and then, after reaching about 10 m/s, remains almost constant.

Draw a tangent to the curve at two different times, $t = 1$ s and $t = 5$ s.

The slope of the lines at 1 s and 5 s is the acceleration at those times.

Calculations:

At 1 s, $a = \dfrac{\text{rise}}{\text{run}} = \dfrac{12.0 \text{ m/s} - 3.0 \text{ m/s}}{2.5 \text{ s} - 0.0 \text{ s}}$;

$a = 3.6 \text{ m/s}^2$

At 5 s, $a = \dfrac{10.7 \text{ m/s} - 10.0 \text{ m/s}}{10 \text{ s} - 0 \text{ s}}$

$a = 0.07 \text{ m/s}^2$

The acceleration is 3.6 m/s^2 at 1 s, and 0.07 m/s^2 at 5 s. It is larger before 1 s and smaller after 5 s. The acceleration is not constant.

Physics & Technology

The Zero Gravity Trainer

The zero-gravity trainer is an aircraft at NASA's Johnson Space Center designed for use in simulating zero-gravity conditions such as those experienced aboard spacecraft orbiting Earth. The plane is a modified Boeing 707 that mimics the free-fall environment aboard a spacecraft as it flies a series of parabola-shaped courses that take the crew from altitudes of about 8000 m up to about 12 000 m and back down again—all in less than two minutes! These short spurts of simulated weightlessness enable the astronauts to practice eating, drinking, and performing a variety of tasks that they will carry out during future missions. Because of the rapid ascents and descents of the aptly nicknamed *Vomit Comet,* training sessions are generally limited to one or two hours.

During a run, the four-engine turbo jet accelerates from 350 knots indicated airspeed (KIA) to about 150 KIA at the top of the parabola. There, the pilot adjusts the jet's engines so that speed is constant. Then, the jet pitches over until the plane is descending again at 350 KIA. During the ascent and descent, acceleration is approximately 1.8 *g.*

Thinking Critically How are the Demon Drop amusement park ride and a ride in the zero-gravity trainer alike?

Positive and Negative Acceleration

You've considered the motion of an accelerating airplane and a sprinter and found that, in both cases, the object's velocity was positive and increasing, and the sign of the acceleration was positive. Now consider a ball being rolled up a slanted driveway. What happens? It slows down, stops briefly, then rolls back down the hill at an increasing speed. Examine the two graphs in **Figure 5–13** that represent the ball's motion and interpret them in the following example problem.
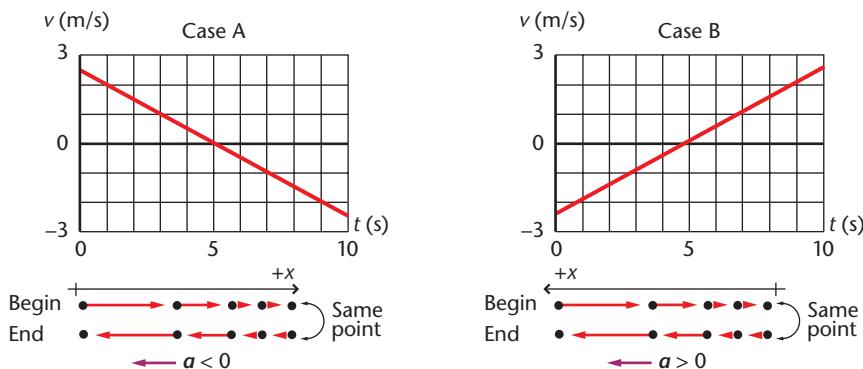

FIGURE 5–13 The sign of the acceleration depends upon the chosen coordinate system.

Example Problem

Finding the Sign of Acceleration

Describe the motion of the ball shown in **Figure 5–13.** What is the difference between the two cases? What is the sign of the ball's acceleration? What is the magnitude of the ball's acceleration?

Strategy:

In each case, the coordinate axis is parallel to the surface. In case A, the ball initially moved in the direction of the positive axis; thus, the sign of the velocity is positive. In case B, the axis was chosen in the opposite direction; thus, the sign of the velocity is negative.

In each case, the ball started with a speed of 2.5 m/s. In both cases, the ball slows down, reaches zero velocity, then speeds up in the opposite direction.

To find the ball's acceleration, use the motion diagrams. Subtract an earlier velocity vector from a later one. Whether the ball is moving uphill or downhill, the acceleration vector points to the left. In case A, the positive axis points to the right, but the acceleration vector points in the opposite direction. Therefore, the acceleration is negative. In case B, the positive axis points to the left, and the acceleration points in the same direction. The acceleration is positive.

Find the magnitude of the acceleration from the slopes of the graphs.

Calculations:

For case A, the ball slows down in the first 5 s.

$\Delta v = v_1 - v_0 = 0.0 \text{ m/s} - 2.5 \text{ m/s} = -2.5 \text{ m/s}$

$a = \Delta v / \Delta t$

$a = (-2.5 \text{ m/s})/(5.0 \text{ s}) = -0.50 \text{ m/s}^2$

During the next 5 s, the ball speeds up in the negative direction.

$\Delta v = -2.5 \text{ m/s} - 0.0 \text{ m/s} = -2.5 \text{ m/s}$

Again, $a = -0.50 \text{ m/s}^2$

Check case B for yourself. Because the axis for case B was chosen in the opposite direction, you will find that a is $+ 0.50 \text{ m/s}^2$, both when the ball is slowing down and when it is speeding up.

Practice Problems

17. An Indy 500 race car's velocity increases from $+4.0$ m/s to $+36$ m/s over a 4.0-s time interval. What is its average acceleration?

18. The race car in problem 17 slows from $+36$ m/s to $+15$ m/s over 3.0 s. What is its average acceleration?

19. A car is coasting backwards downhill at a speed of 3.0 m/s when the driver gets the engine started. After 2.5 s, the car is moving uphill at 4.5 m/s. Assuming that uphill is the positive direction, what is the car's average acceleration?

20. A bus is moving at 25 m/s when the driver steps on the brakes and brings the bus to a stop in 3.0 s.

a. What is the average acceleration of the bus while braking?

b. If the bus took twice as long to stop, how would the acceleration compare with what you found in part **a?**

21. Look at the v-t graph of the toy train in **Figure 5–14.**

a. During which time interval or intervals is the speed constant?

b. During which interval or intervals is the train's acceleration positive?

c. During which time interval is its acceleration most negative?

22. Using **Figure 5–14,** find the average acceleration during the following time intervals.

**a.** 0 to 5 s   **b.** 15 to 20 s   **c.** 0 to 40 s
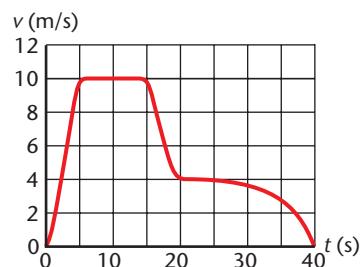
FIGURE 5–14

Acceleration when instantaneous velocity is zero What happens to the acceleration when $v = 0$, that is, when the ball in the example problem stops and reverses direction? Consider the motion diagram that starts when the ball is still moving uphill and ends when the ball is

Uphill
Begin

At top $v = 0$

Downhill
End
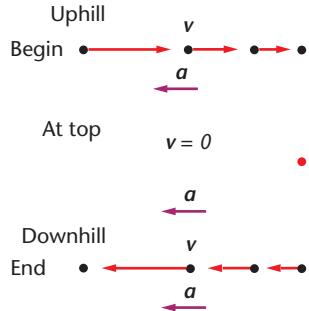
FIGURE 5–15 The acceleration of the ball is always downward.

moving back downhill as in **Figure 5–15.** The acceleration is downhill both before and after the ball stops. What happens at the instant when the ball's instantaneous velocity is zero? Remember that the velocity is zero only *at* an instant of time, not *for* an instant. That is, the time inter-val over which the velocity is zero is itself zero. Thus, the acceleration points downhill as the ball reaches the top of the hill, and it continues to point downhill as the ball moves back downhill.

Calculating Velocity from Acceleration

You learned that you could use the definition of velocity to find the position of an object moving at constant velocity. In the same way, you can find the velocity of the object by rearranging the definition of aver-age acceleration.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_1 - v_0}{t_1 - t_0}$$

Assume that $t_0 = 0$. At that time, the object has a velocity v_0. Let t be any value of t_1 and v be the value of the velocity at that time. Because only one-dimensional, straight-line motion with constant acceleration will be considered, a can be used instead of $\bar{a}$. After making these sub-stitutions and rearranging the equation, the following equation is obtained for the velocity of an object moving at constant acceleration.

Velocity with Constant Acceleration $v = v_0 + at$

You also can use this equation to find the time at which a constantly accelerating object has a given velocity, or, if you are given both a velocity and the time at which it occurred, you can calculate the initial velocity.

Practice Problems

23. A golf ball rolls up a hill toward a miniature-golf hole. Assign the direction toward the hole as being positive.
    a. If the ball starts with a speed of 2.0 m/s and slows at a con-stant rate of 0.50 m/s^2, what is its velocity after 2.0 s?
    b. If the constant acceleration continues for 6.0 s, what will be its velocity then?
    c. Describe in words and in a motion diagram the motion of the golf ball.
24. A bus, traveling at 30.0 km/h, speeds up at a constant rate of 3.5 m/s^2. What velocity does it reach 6.8 s later?
25. If a car accelerates from rest at a constant 5.5 m/s^2, how long will it need to reach a velocity of 28 m/s?
26. A car slows from 22 m/s to 3.0 m/s at a constant rate of 2.1 m/s^2. How many seconds are required before the car is traveling at 3.0 m/s?

CONTENTS

Displacement Under Constant Acceleration

You know how to use the area under the curve of a velocity-time graph to find the displacement when the velocity is constant. The same method can be used to find the displacement when the acceleration is constant. **Figure 5–16** is a graph of the motion of an object accelerating constantly from v_0 to v. If the velocity had been a constant, v_0, the displacement would have been $v_0 t$, the area of the lightly shaded rectangle. Instead, the velocity increased from v_0 to v. Thus, the displacement is increased by the area of the triangle, $1/2(v - v_0)t$. The total displacement, then, is the sum of the two.

$$d = v_0 t + 1/2(v - v_0)t$$

When the terms are combined, the following equation results.

$$d = 1/2(v + v_0)t$$

If the initial position, d_0, is not zero, then this term must be added to give the general equation for the final position.

Final Position with Constant Acceleration $\quad d = d_0 + 1/2(v + v_0)t$

Frequently, the velocity at time t is not known, but because $v = v_0 + at$, you can substitute $v_0 + at$ for v in the previous equation and obtain the following equation.

$$d = d_0 + 1/2(v_0 + v_0 + at)t$$

When the terms are combined, the following equation results.

Final Position with Constant Acceleration $\quad d = d_0 + v_0 t + 1/2at^2$

Note that the third equation involves position, velocity, and time, but not acceleration. The fifth equation involves position, acceleration, and time, but not velocity. Is there an equation that relates position, velocity, and acceleration, but doesn't include time? To find that equation, start with the following equations.

$$d = d_0 + 1/2(v + v_0)t \text{ and } v = v_0 + at$$

Solve the second equation for t.

$$t = (v - v_0)/a$$

Substitute this into the equation for displacement.

$$d = d_0 + 1/2(v_0 + v)(v - v_0)/a$$

This equation can be solved for the final velocity.

Final Velocity with Constant Acceleration $\quad v^2 = v_0{}^2 + 2a(d - d_0)$

The four equations that have been derived for motion under constant acceleration are summarized in **Table 5–2.** One is useful for calculating velocity and three are equations for position. When solving problems involving constant acceleration, determine what information is given and what is unknown, then choose the appropriate equation. These equations, along with velocity-time and position-time graphs, provide the mathematical models you need to solve motion problems.
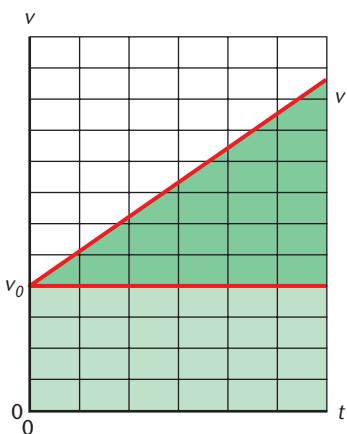
FIGURE 5–16 The area under the curve of a velocity versus time graph equals the displacement.

Pocket Lab

Direction of Acceleration

Tape a bubble level onto the top of a laboratory cart. Center the bubble. Observe the direction of the motion of the bubble as you pull the cart forward, move it at constant speed, and allow it to coast to a stop. Relate the motion of the bubble to the acceleration of the cart. Predict what would happen if you tie the string to the back of the cart and repeat the experiment. Try it.

Analyze and Conclude Draw motion diagrams for the cart as you moved it in the forward direction and it coasted to a stop and as you repeated the experiment in the opposite direction.

Ball and Car Race

Problem

A car moving along a highway passes a parked police car with a radar detector. Just as the car passes, the police car starts to pursue, moving with a constant acceleration. The police car catches up with the car just as it leaves the jurisdiction of the policeman.

Hypothesis

Sketch the position-versus-time graphs and the velocity-versus-time graphs for this chase, then simulate the chase.
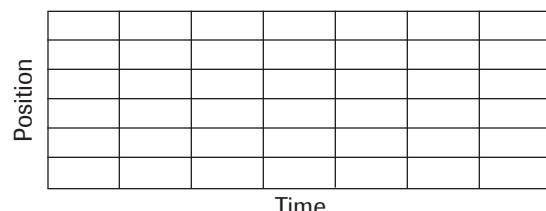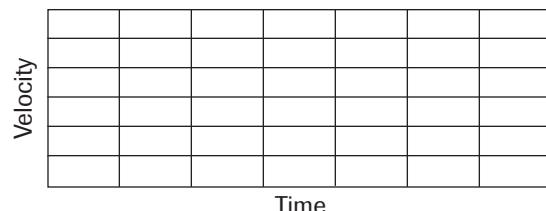
Possible Materials

battery-powered car	1-in. steel ball
masking tape	stopwatch
wood block	graph paper
90-cm-long grooved track	

Plan the Experiment

1. Identify the variables in this activity.
2. Determine how you will give the ball a constant acceleration.
3. Devise a method to ensure that both objects reach the end of the track at the same time.
4. Construct a data table that will show the positions of both objects at the beginning, the halfway point, and the end of the chase.
5. **Check the Plan** Review your plan with your teacher before you begin the race.
6. Construct *p-t* and *v-t* graphs for both objects. Use technology to construct these graphs if possible. Identify the relationships between variables.
7. Dispose of materials that cannot be reused or recycled. Put away materials that can be used again.

[velocity vs. time graph]

Velocity

Time

[position vs. time graph]

Position

Time

Analyze and Conclude

1. **Comparing and Contrasting** Compare the velocities of the cars at the beginning and at the end of the chase. Write a verbal description.
2. **Using Graphs** At any time during the chase, did the cars ever have the same velocity? If so, mark these points on the graphs.
3. **Comparing and Contrasting** Compare the average velocity of the police car to that of the car.
4. **Calculating Results** Calculate the average speed of each car.

Apply

1. Explain why it took the police car so long to catch the car after it sped by.
2. Analyze and evaluate the plots of the speeder's motion. Infer from the plots the speeder's acceleration.
3. If the speeder accelerated at the exact same rate of the police car at the moment the speeder passed the police car, would the police car ever catch the speeder? Predict how your graphs would change.
4. Develop a CBL lab that plots the velocity of a non-accelerated object and an accelerated object. Describe your graphs.

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TABLE 5–2		
Equations of Motion for Uniform Acceleration		
Equation	**Variables**	**Initial Conditions**
$v = v_0 + at$	$t \quad v \quad a$	v_0
$d = d_0 + 1/2(v_0 + v)t$	$t \quad d \quad v$	$d_0 \quad v_0$
$d = d_0 + v_0 t + 1/2at^2$	$t \quad d \quad a$	$d_0 \quad v_0$
$v^2 = v_0^2 + 2a(d - d_0)$	$d \quad v \quad a$	$d_0 \quad v_0$

Example Problem

Finding Displacement Under Constant Acceleration

In Chapter 3, you completed the first step in solving the following problem by sketching the situation and drawing the motion diagram. Now you can add the mathematical model. A car starts at rest and speeds up at 3.5 m/s² after the traffic light turns green. How far will it have gone when it is going 25 m/s?

Sketch the Problem

- Sketch the situation.
- Establish coordinate axes.
- Draw a motion diagram.

Calculate Your Answer

Known:

$d_0 = 0.0$ m $\quad v = 25$ m/s

$v_0 = 0.0$ m/s $\quad a = 3.5$ m/s²

Unknown:

$d = ?$
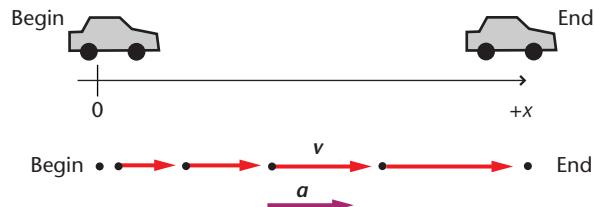
Strategy:

Refer to **Table 5–2.** Use an equation containing v, a, and d.

Calculations:

$v^2 = v_0^2 + 2a(d - d_0)$

$d = d_0 + (v^2 - v_0^2)/(2a)$

$\quad = 0.0$ m $+ [(25$ m/s$)^2 - (0.0$ m/s$)^2]/(2 \cdot 3.5$ m/s²$)$

$\quad = 89$ m

Check Your Answer

- Is the unit correct? Dividing m²/s² by m/s² results in m, the correct unit for position.
- Does the sign make sense? It is positive, in agreement with both the pictorial and physical models.
- Is the magnitude realistic? The displacement is almost the length of a football field. It seems large, but 25 m/s is fast (about 55 mph), and the acceleration, as you will find in the next example problem, is not very great. Therefore, the result is reasonable.

Two-Part Motion

The driver of the car in the previous example problem, traveling at a constant 25 m/s, sees a child suddenly run into the road. It takes the driver 0.45 s to hit the brakes. As it slows, the car has a steady acceleration of 8.5 m/s^2. What's the total distance the car moves before it stops?

Sketch the Problem

- Label your drawing with "begin" and "end."
- Choose a coordinate system and create the motion diagram.
- Use subscripts to distinguish the three positions in the problem.

Calculate Your Answer

Known:

$d_1 = 0.0$ m $v_2 = 25$ m/s

$v_1 = 25$ m/s $a_{23} = -8.5$ m/s^2

$a_{12} = 0.0$ m/s^2 $v_3 = 0.0$ m/s

$t_2 = 0.45$ s

Unknown:

$d_2 = ?$

$d_3 = ?$
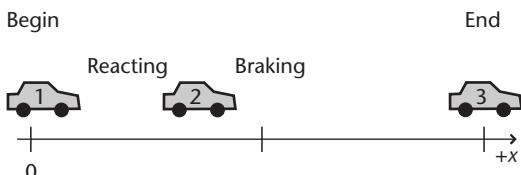
Strategy:

There are two parts to the problem: the interval of reacting and the interval of braking.

Reacting: Find the distance the car travels. During this time, the velocity and time are known and the velocity is constant.

Braking: Find the distance the car moves while braking. The initial and final velocities are known. The acceleration is constant and negative, as shown in the motion diagram.

The position of the car when the brakes are applied, d_2, is the solution of the first part of the problem; it is needed to solve the second part.

Calculations:

Reacting: $d_2 = vt$

$d_2 = (25 \text{ m/s})(0.45 \text{ s})$

$= 11$ m

Braking: $v_3{}^2 = v_2{}^2 + 2a_{23}(d_3 - d_2)$

$d_3 = d_2 + (v_3{}^2 - v_2{}^2)/(2a_{23})$

$= 11 \text{ m} + \dfrac{0.0 - (25 \text{ m/s})^2}{2(-8.5 \text{ m/s}^2)}$

$= 48$ m

Check Your Answer

- Is the unit correct? Performing algebra on the units verifies the distance in meters.
- Do the signs make sense? Both d_3 and d_2 are positive, as they should be.
- Is the magnitude realistic? The braking distance is much smaller than it was in the previous example problem, which makes sense because the magnitude of the acceleration is larger.

For all problems, sketch the situation, assign variables, create a motion diagram, and then develop a mathematical model.

27. A race car traveling at 44 m/s slows at a constant rate to a velocity of 22 m/s over 11 s. How far does it move during this time?

28. A car accelerates at a constant rate from 15 m/s to 25 m/s while it travels 125 m. How long does it take to achieve this speed?

29. A bike rider accelerates constantly to a velocity of 7.5 m/s during 4.5 s. The bike's displacement during the acceleration was 19 m. What was the initial velocity of the bike?

30. An airplane starts from rest and accelerates at a constant 3.00 m/s^2 for 30.0 s before leaving the ground.
   a. How far did it move?
   b. How fast was it going when it took off?

5.3 Section Review

1a. Give an example of an object that is slowing down but has a positive acceleration.

 b. Give an example of an object that is speeding up but has a negative acceleration.

2a. If an object has zero acceleration, does that mean its velocity is zero? Give an example.

 b. If an object has zero velocity at some instant, does that mean its acceleration is zero? Give an example.

3. Is km/h/s a unit of acceleration? Is this unit the same as km/s/h? Explain.

4. **Figure 5–17** is a strobe photo of a horizontally moving ball. What information about the photo would you need and what measurements would you make to estimate the acceleration?

5. If you are given a table of velocities of an object at various times, how could you find out if the acceleration was constant?
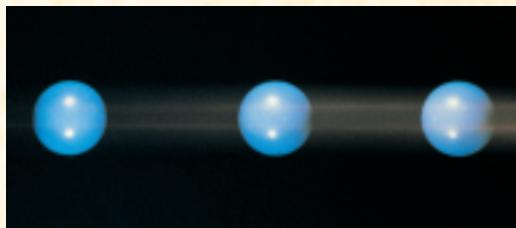
FIGURE 5–17

6. If you are given initial and final velocities and the constant acceleration of an object, and you are asked to find the displacement, which equation would you use?

7. **Critical Thinking** Describe how you could calculate the acceleration of an automobile. Specify the measuring instruments and the procedures you would use.

5.4

Free Fall

Drop a sheet of paper. Crumple it, then drop it again. Its motion is different in the two instances. So is the motion of a pebble falling through water compared with the same pebble falling through air. Do heavier objects fall faster than lighter ones? The answer depends upon whether you drop sheets of paper or rocks.

OBJECTIVES

- **Recognize** the meaning of the acceleration due to gravity.

- **Define** the magnitude of the acceleration due to gravity as a positive quantity and **determine** the sign of the acceleration relative to the chosen coordinate system.

- **Use** the motion equations to solve problems involving freely falling objects.

Acceleration Due to Gravity
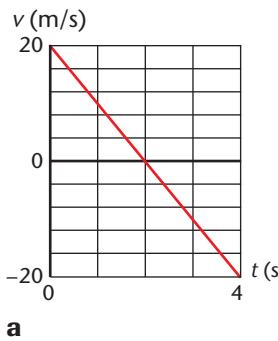
Galileo Galilei recognized about 400 years ago that to make progress in the study of the motion of falling bodies, the effects of air or water, the medium through which the object falls, had to be ignored. He also knew that he had no means of recording the fall of objects, so he rolled balls down inclined planes. By "diluting" gravity in this way, he could make careful measurements even with simple instruments.

Galileo found that, neglecting the effect of the air, all freely falling objects had the same acceleration. It didn't matter what they were made of, what their masses were, from how high they were dropped, or whether they were dropped or thrown. The magnitude of the acceleration of falling objects is given a special symbol, g, equal to 9.80 m/s^2. We now know that there are small variations in g at different places on Earth, and that 9.80 m/s^2 is the average value.
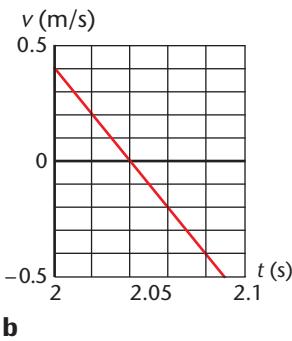
Note that g is a positive quantity. You will never use a negative value of g in a problem. But don't things accelerate downward, and isn't down usually the negative direction? Although this is true, remember that g is only the magnitude of the acceleration, not the acceleration itself. If upward is defined to be the positive direction, then the acceleration due to gravity is equal to $-g$. The **acceleration due to gravity** is the acceleration of an object in free fall that results from the influence of Earth's gravity. Suppose you drop a rock. One second later, its velocity is 9.80 m/s downward. One second after that, its velocity is 19.60 m/s downward. For each second that the rock is falling, its downward velocity increases by 9.80 m/s.

Look at the strobe photo of a dropped apple in **Figure 5–18.** The time interval between the photos is 1/120 s. The displacement between each pair of images increases, so the speed is increasing. If the upward direction is chosen as positive, then the velocity is becoming more and more negative.
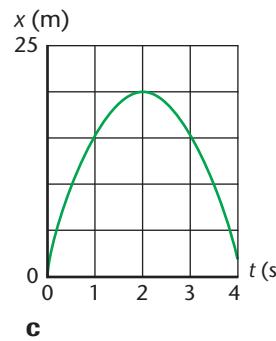
Could this photo be of a ball thrown upward? If you again choose upward as the positive direction, then the ball leaves your hand with a positive velocity of, say, 20.0 m/s. The acceleration is downward, so a is negative. That is, $a = -g = -9.80$ m/s^2. This means that the speed of the ball becomes less and less, which is in agreement with the strobe

FIGURE 5–18 If the upward direction is chosen as positive, then both the velocity and the acceleration of this apple in free fall are negative.

a

b

c

d

FIGURE 5–19 In a coordinate system in which the upward direction is positive, the velocity of the thrown ball decreases until it becomes zero at 2.04 s, then it increases in the negative direction.
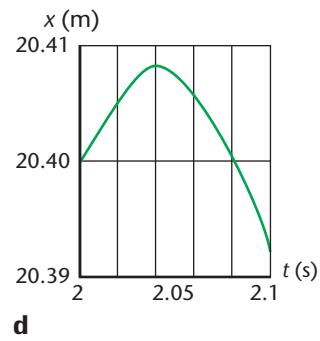
photo. After 1 s, the ball's velocity is reduced by 9.80 m/s, so it is now traveling at 10.2 m/s. After 2 s, the velocity is 0.4 m/s, and the ball is still moving upward. After the next second, the ball's velocity, being reduced by another 9.80 m/s, is now −9.4 m/s. The ball is now moving downward. After 4 s, the velocity is −19.2 m/s, meaning that it is falling even faster. The velocity-time graph of the ball's flight is shown in **Figure 5–19a. Figure 5–19b** shows what happens at around 2 s, where the velocity changes smoothly from positive to negative. At an instant of time, near 2.04 s, the ball's velocity is zero.

The position-time graphs in **Figure 5–19c** and **d** show how the ball's height changes. The ball has its maximum height when its velocity is zero.

Free Fall

Example Problem

The Demon Drop

The Demon Drop ride at Cedar Point Amusement Park falls freely for 1.5 s after starting from rest.

a. What is its velocity at the end of 1.5 s?

b. How far does it fall?

Sketch the Problem

- Choose a coordinate system with a positive axis upward and the origin at the initial position of the car.
- Label "begin" and "end."
- Draw a motion diagram showing that both **a** and **v** are downward and, therefore, negative.

Calculate Your Answer

Known:

$a = -g = -9.80 \text{ m/s}^2$

$d_0 = 0$

$v_0 = 0$

$t = 1.5 \text{ s}$

Unknown:

$d = ?$

$v = ?$

Continued on next page

CONTENTS

Strategy:

a. Use the equation for velocity at constant acceleration.

b. Use the equation for displacement when time and constant acceleration are known.

Calculations:

$v = v_0 + at$

$v = 0 + (-9.80 \text{ m/s}^2)(1.5 \text{ s}) = -15 \text{ m/s}$

$d = d_0 + v_0 t + 1/2 a t^2$

$d = 0 + 0 + 1/2(-9.80 \text{ m/s}^2)(1.5 \text{ s})^2 = -11 \text{ m}$

Check Your Answer

- Are the units correct? Performing algebra on the units verifies velocity in m/s and position in m.
- Do the signs make sense? Negative signs agree with the diagram.
- Are the magnitudes realistic? Yes, when judged by the photo at the opening of this chapter in which the car is about the height of a person, about 2 m.

Practice Problems

31. A brick is dropped from a high scaffold.
  a. What is its velocity after 4.0 s?
  b. How far does the brick fall during this time?
32. A tennis ball is thrown straight up with an initial speed of 22.5 m/s. It is caught at the same distance above ground.
  a. How high does the ball rise?
  b. How long does the ball remain in the air? (Hint: The time to rise equals the time to fall. Can you show this?)
33. A spaceship far from any star or planet accelerates uniformly from 65.0 m/s to 162.0 m/s in 10.0 s. How far does it move?

5.4 Section Review

1. Gravitational acceleration on Mars is about 1/3 that on Earth. Suppose you could throw a ball upward with the same velocity on Mars as on Earth.
   a. How would the ball's maximum height compare to that on Earth?
   b. How would its flight time compare?
2. Research and describe Galileo's contributions to physics.

3. **Critical Thinking** When a ball is thrown vertically upward, it continues upward until it reaches a certain position, then it falls down again. At that highest point, its velocity is instantaneously zero. Is the ball accelerating at the highest point? Devise an experiment to prove or disprove your answer.

CHAPTER **5** REVIEW

Summary

5.1 Graphing Motion in One Dimension

• Position-time graphs can be used to find the velocity and position of an object, and where and when two objects meet.
• A description of motion can be obtained by interpreting graphs, and graphs can be drawn from descriptions of motion.
• Equations that describe the position of an object moving at constant velocity can be written based on word and graphical representations of problems.

5.2 Graphing Velocity in One Dimension

• Instantaneous velocity is the slope of the tangent to the curve on a position-time graph.
• Velocity-time graphs can be used to determine the velocity of an object and the time when two objects have the same velocity.
• The area under the curve on a velocity-time graph is displacement.

5.3 Acceleration

• The acceleration of an object is the slope of the curve on a velocity-time graph.

• The slope of the tangent to the curve on a v-t graph is the instantaneous acceleration of the object.
• Velocity-time graphs and motion diagrams can be used to find the sign of the acceleration.
• Both graphs and equations can be used to find the velocity of an object undergoing constant acceleration.
• Three different equations give the displacement of an object under constant acceleration, depending on what quantities are known.
• The mathematical model completes the solution of motion problems.
• Results obtained by solving a problem must be tested to find out whether they are reasonable.

5.4 Free Fall

• The magnitude of the acceleration due to gravity ($g = 9.80$ m/s^2) is always a positive quantity. The sign of acceleration depends upon the choice of the coordinate system.
• Motion equations can be used to solve problems involving freely falling objects.

Key Equations

5.1

$$\bar{v} = \frac{\Delta d}{\Delta t} = \frac{d_1 - d_0}{t_1 - t_0}$$

$$d = d_0 + vt$$

5.3

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_1 - v_0}{t_1 - t_0} \qquad d = d_0 + v_0 t + 1/2 a t^2$$

$$v = v_0 + at \qquad\qquad v^2 = v_0{}^2 + 2a(d - d_0)$$

$$d = d_0 + 1/2(v + v_0)t$$

Reviewing Concepts

Section 5.1

1. A walker and a runner leave your front door at the same time. They move in the same direction at different constant velocities. Describe the position-time graphs of each.

CONTENTS

2. What does the slope of the tangent to the curve on a position-time graph measure?

Section 5.2

3. If you know the positions of an object at two points along its path, and you also know the time it took to get from one point to the other, can you determine the particle's instantaneous velocity? Its average velocity? Explain.

4. What quantity is represented by the area under a velocity-time curve?

5. **Figure 5–20** shows the velocity-time graph for an automobile on a test track. Describe how the velocity changes with time.
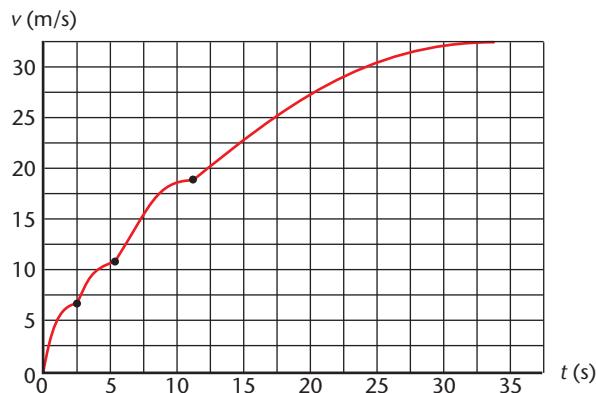
FIGURE 5–20

Section 5.3

6. What does the slope of the tangent to the curve on a velocity-time graph measure?

7. A car is traveling on an interstate highway.
  a. Can the car have a negative velocity and a positive acceleration at the same time? Explain.
  b. Can the car's velocity change signs while it is traveling with constant acceleration? Explain.

8. Can the velocity of an object change when its acceleration is constant? If so, give an example. If not, explain.

9. If the velocity-time curve is a straight line parallel to the t-axis, what can you say about the acceleration?

10. If you are given a table of velocities of an object at various times, how could you find out if the acceleration of the object is constant?

11. Write a summary of the equations for position, velocity, and time for an object experiencing uniformly accelerated motion.

Section 5.4

12. Explain why an aluminum ball and a steel ball of similar size and shape, dropped from the same height, reach the ground at the same time.

13. Give some examples of falling objects for which air resistance cannot be ignored.

14. Give some examples of falling objects for which air resistance can be ignored.

Applying Concepts

15. **Figure 5–20** shows the velocity-time graph of an accelerating car. The three "notches" in the curve occur where the driver changes gears.
  a. Describe the changes in velocity and acceleration of the car while in first gear.
  b. Is the acceleration just before a gear change larger or smaller than the acceleration just after the change? Explain your answer.

16. Explain how you would walk to produce each of the position-time graphs in **Figure 5–21.**
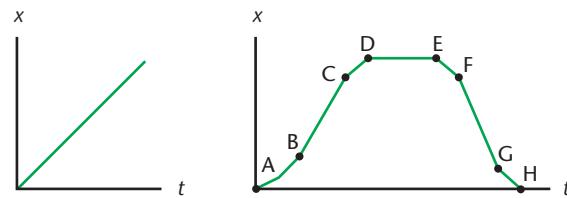
FIGURE 5–21

17. Use **Figure 5–20** to determine during what time interval the acceleration is largest and during what time interval the acceleration is smallest.

18. Solve the equation $v = v_0 + at$ for acceleration.

19. **Figure 5–22** is a position-time graph of two people running.
  a. Describe the position of runner A relative to runner B at the y-intercept.
  b. Which runner is faster?
  c. What occurs at point P and beyond?

CONTENTS
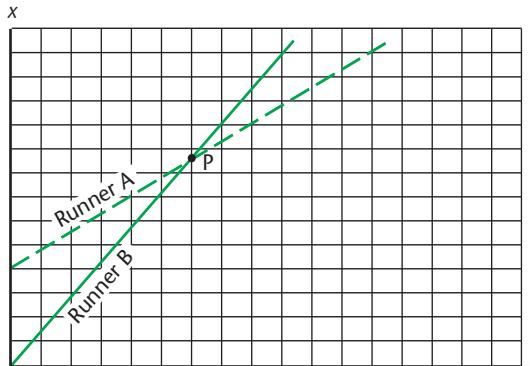
FIGURE 5–22
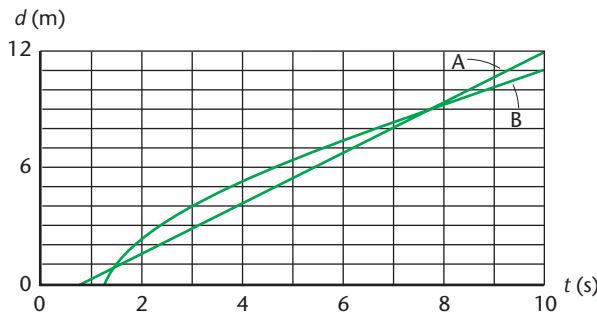
FIGURE 5–23

FIGURE 5–24

20. Figure 5–23 is a position-time graph of the motion of two cars on a road.
  a. At what time(s) does one car pass the other?
  b. Which car is moving faster at 7.0 s?
  c. At what time(s) do the cars have the same velocity?
  d. Over what time interval is car B speeding up all the time?
  e. Over what time interval is car B slowing down all the time?
21. Look at **Figure 5–24.**
  a. What kind of motion is represented by **a?**
  b. What does the area under the curve represent?
  c. What kind of motion is represented by **b?**
  d. What does the area under the curve represent?
22. An object shot straight up rises for 7.0 s before it reaches its maximum height. A second object falling from rest takes 7.0 s to reach the ground. Compare the displacements of the two objects during this time interval.
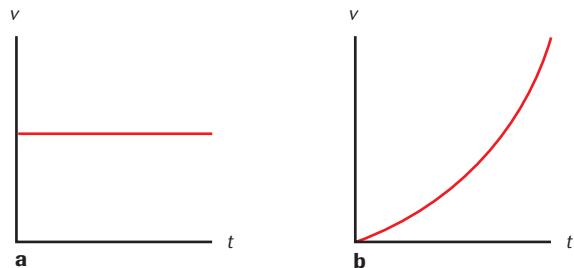
23. Describe the changes in the velocity of a ball thrown straight up into the air. Then describe the changes in the ball's acceleration.
24. The value of g on the moon is 1/6 of its value on Earth.
  a. Will a ball dropped by an astronaut hit the surface of the moon with a smaller, equal, or larger speed than that of a ball dropped from the same height to Earth?
  b. Will it take more, less, or equal time to fall?
25. Planet Dweeb has three times the gravitational acceleration of Earth. A ball is thrown vertically upward with the same initial velocity on Earth and on Dweeb.
 **a.** How does the maximum height reached by the ball on Dweeb compare to the maximum height on Earth?
  b. If the ball on Dweeb were thrown with three times greater initial velocity, how would that affect your answer to **a?**
26. Rock A is dropped from a cliff; rock B is thrown upward from the same position.
  a. When they reach the ground at the bottom of the cliff, which rock has a greater velocity?
  b. Which has a greater acceleration?
  c. Which arrives first?

Problems
Section 5.1
27. Light from the sun reaches Earth in 8.3 min. The velocity of light is 3.00×10^8 m/s. How far is Earth from the sun?
28. You and a friend each drive 50.0 km. You travel at 90 km/h; your friend travels at 95.0 km/h. How long will your friend wait for you at the end of the trip?

29. The total distance a steel ball rolls down an incline at various times is given in **Table 5–3.**
   a. Draw a position-time graph of the motion of the ball. When setting up the axes, use five divisions for each 10 m of travel on the d-axis. Use five divisions for 1 s of time on the t-axis.
   b. What type of curve is the line of the graph?
   c. What distance has the ball rolled at the end of 2.2 s?

TABLE 5–3	
Distance versus Time	
Time (s)	Distance (m)
0.0	0.0
1.0	2.0
2.0	8.0
3.0	18.0
4.0	32.9
5.0	50.0

30. A cyclist maintains a constant velocity of +5.0 m/s. At time $t = 0.0$, the cyclist is +250 m from point A.
   a. Plot a position-time graph of the cyclist's location from point A at 10.0-s intervals for 60.0 s.
   b. What is the cyclist's position from point A at 60.0 s?
   c. What is the displacement from the starting position at 60.0 s?

31. From the position-time graph in **Figure 5–25,** construct a table showing the average velocity of the object during each 10-s interval over the entire 100 s.

x (m)
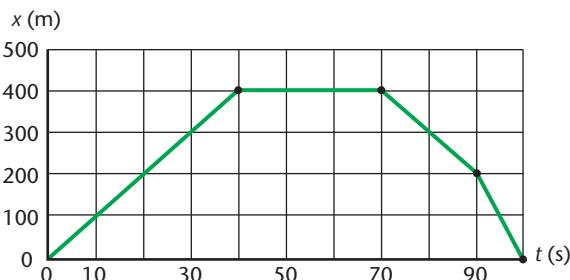
FIGURE 5–25

32. Plot the data in **Table 5–4** on a position-time graph. Find the average velocity in the time interval between 0.0 s and 5.0 s.

TABLE 5–4	
Position versus Time	
Clock Reading, t (s)	Position, d (m)
0.0	30
1.0	30
2.0	35
3.0	45
4.0	60
5.0	70

33. You drive a car for 2.0 h at 40 km/h, then for another 2.0 h at 60 km/h.
   a. What is your average velocity?
   b. Do you get the same answer if you drive 1.0×10^2 km at each of the two speeds?

34. Use the position-time graph in **Figure 5–25** to find how far the object travels
   a. between $t = 0$ s and $t = 40$ s.
   b. between $t = 40$ s and $t = 70$ s.
   c. between $t = 90$ s and $t = 100$ s.

35. Do this problem on a worksheet. Both car A and car B leave school when a clock reads zero. Car A travels at a constant 75 km/h, and car B travels at a constant 85 km/h.
   a. Draw a position-time graph showing the motion of both cars.
   b. How far are the two cars from school when the clock reads 2.0 h? Calculate the distances using the equation for motion and show them on your graph.
   c. Both cars passed a gas station 120 km from the school. When did each car pass the gas station? Calculate the times and show them on your graph.

36. Draw a position-time graph for two cars driving to the beach, which is 50 km from school. At noon Car A leaves a store 10 km closer to the beach than the school is and drives at 40 km/h. Car B starts from school at 12:30 P.M. and drives at 100 km/h. When does each car get to the beach?

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37. Two cars travel along a straight road. When a stopwatch reads $t = 0.00$ h, car A is at $d_A = 48.0$ km moving at a constant 36.0 km/h. Later, when the watch reads $t = 0.50$ h, car B is at $d_B = 0.00$ km moving at 48.0 km/h. Answer the following questions, first, graphically by creating a position-time graph, and second, algebraically by writing down equations for the positions d_A and d_B as a function of the stopwatch time, t.
   a. What will the watch read when car B passes car A?
   b. At what position will car B pass car A?
   c. When the cars pass, how long will it have been since car A was at the reference point?

38. A car is moving down a street at 55 km/h. A child suddenly runs into the street. If it takes the driver 0.75 s to react and apply the brakes, how many meters will the car have moved before it begins to slow down?

Section 5.2

39. Refer to **Figure 5–23** to find the instantaneous speed for
   a. car B at 2.0 s.
   b. car B at 9.0 s.
   c. car A at 2.0 s.

40. Refer to **Figure 5–26** to find the distance the moving object travels between
   a. $t = 0$ s and $t = 5$ s.
   b. $t = 5$ s and $t = 10$ s.
   c. $t = 10$ s and $t = 15$ s.
   d. $t = 0$ s and $t = 25$ s.
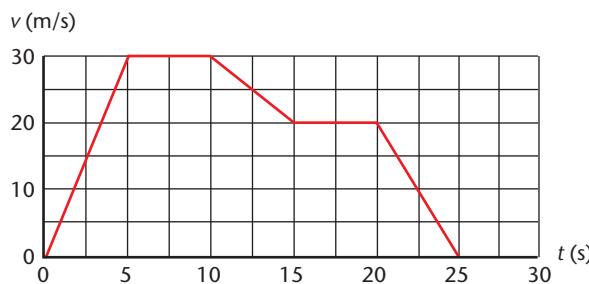
FIGURE 5–26

41. Find the instantaneous speed of the car in **Figure 5–20** at 15 s.

42. You ride your bike for 1.5 h at an average velocity of 10 km/h, then for 30 min at 15 km/h. What is your average velocity?

43. Plot a velocity-time graph using the information in **Table 5–5,** then answer the questions.
   a. During what time interval is the object speeding up? Slowing down?
   b. At what time does the object reverse direction?
   c. How does the average acceleration of the object in the interval between 0 s and 2 s differ from the average acceleration in the interval between 7 s and 12 s?

TABLE 5–5			
Velocity versus Time			
Time (s)	**Velocity (m/s)**	**Time (s)**	**Velocity (m/s)**
0.0	4.0	7.0	12.0
1.0	8.0	8.0	8.0
2.0	12.0	9.0	4.0
3.0	14.0	10.0	0.0
4.0	16.0	11.0	−4.0
5.0	16.0	12.0	−8.0
6.0	14.0		

Section 5.3

44. Find the uniform acceleration that causes a car's velocity to change from 32 m/s to 96 m/s in an 8.0-s period.

45. Use **Figure 5–26** to find the acceleration of the moving object
   a. during the first 5 s of travel.
   b. between the fifth and the tenth second of travel.
   c. between the tenth and the 15th second of travel.
   d. between the 20th and 25th second of travel.

46. A car with a velocity of 22 m/s is accelerated uniformly at the rate of 1.6 m/s^2 for 6.8 s. What is its final velocity?

47. A supersonic jet flying at 145 m/s is accelerated uniformly at the rate of 23.1 m/s^2 for 20.0 s.
   a. What is its final velocity?
   b. The speed of sound in air is 331 m/s. How many times the speed of sound is the plane's final speed?

48. Determine the final velocity of a proton that has an initial velocity of 2.35×10^5 m/s, and then is accelerated uniformly in an electric field at the rate of -1.10×10^{12} m/s^2 for 1.50×10^{-7} s.

49. Determine the displacement of a plane that is uniformly accelerated from 66 m/s to 88 m/s in 12 s.

50. How far does a plane fly in 15 s while its velocity is changing from 145 m/s to 75 m/s at a uniform rate of acceleration?

51. A car moves at 12 m/s and coasts up a hill with a uniform acceleration of -1.6 m/s^2.
a. How far has it traveled after 6.0 s?
b. How far has it gone after 9.0 s?

52. A plane travels 5.0×10^2 m while being accelerated uniformly from rest at the rate of 5.0 m/s^2. What final velocity does it attain?

53. A race car can be slowed with a constant acceleration of -11 m/s^2.
a. If the car is going 55 m/s, how many meters will it take to stop?
b. How many meters will it take to stop a car going twice as fast?

54. An engineer must design a runway to accommodate airplanes that must reach a ground velocity of 61 m/s before they can take off. These planes are capable of being accelerated uniformly at the rate of 2.5 m/s^2.
a. How long will it take the planes to reach takeoff speed?
b. What must be the minimum length of the runway?

55. Engineers are developing new types of guns that might someday be used to launch satellites as if they were bullets. One such gun can give a small object a velocity of 3.5 km/s, moving it through only 2.0 cm.
a. What acceleration does the gun give this object?
b. Over what time interval does the acceleration take place?

56. Highway safety engineers build soft barriers so that cars hitting them will slow down at a safe rate. A person wearing a seat belt can withstand an acceleration of -3.0×10^2 m/s^2. How thick should barriers be to safely stop a car that hits a barrier at 110 km/h?

57. A baseball pitcher throws a fastball at a speed of 44 m/s. The acceleration occurs as the pitcher holds the ball in his hand and moves it through an almost straight-line distance of 3.5 m. Calculate the acceleration, assuming it

is uniform. Compare this acceleration to the acceleration due to gravity, 9.80 m/s^2.

58. Rocket-powered sleds are used to test the responses of humans to acceleration. Starting from rest, one sled can reach a speed of 444 m/s in 1.80 s and can be brought to a stop again in 2.15 s.
a. Calculate the acceleration of the sled when starting, and compare it to the magnitude of the acceleration due to gravity, 9.80 m/s^2.
b. Find the acceleration of the sled when braking and compare it to the magnitude of the acceleration due to gravity.

59. Draw a velocity-time graph for each of the graphs in **Figure 5–27.**
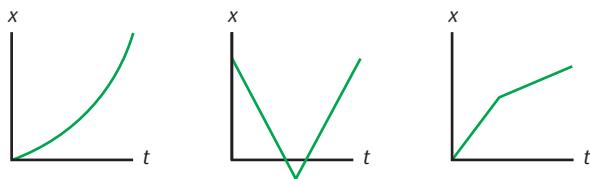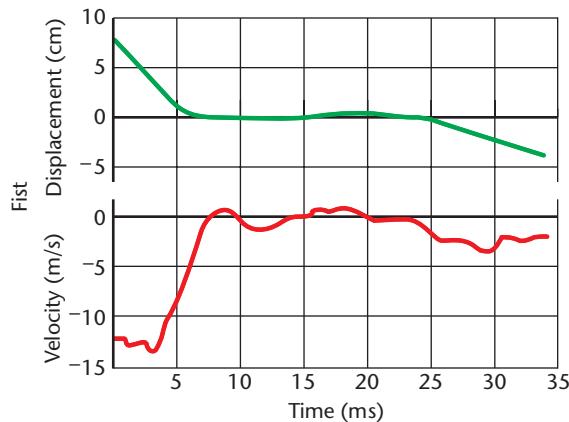
FIGURE 5–27

60. The velocity of an automobile changes over an 8.0-s time period as shown in **Table 5–6.**
a. Plot the velocity-time graph of the motion.
b. Determine the displacement of the car during the first 2.0 s.
c. What displacement does the car have during the first 4.0 s?
d. What displacement does the car have during the entire 8.0 s?
e. Find the slope of the line between $t = 0.0$ s and $t = 4.0$ s. What does this slope represent?
f. Find the slope of the line between $t = 5.0$ s and $t = 7.0$ s. What does this slope indicate?

TABLE 5–6			
Velocity versus Time			
Time (s)	**Velocity (m/s)**	**Time (s)**	**Velocity (m/s)**
0.0	0.0	5.0	20.0
1.0	4.0	6.0	20.0
2.0	8.0	7.0	20.0
3.0	12.0	8.0	20.0
4.0	16.0		

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FIGURE 5-28

61. Figure 5–28 shows the position-time and velocity-time graphs of a karate expert's fist as it breaks a wooden board.
 **a.** Use the velocity-time graph to describe the motion of the expert's fist during the first 10 ms.
  b. Estimate the slope of the velocity-time graph to determine the acceleration of the fist when it suddenly stops.
  c. Express the acceleration as a multiple of the gravitational acceleration, $g = 9.80$ m/s^2.
  d. Determine the area under the velocity-time curve to find the displacement of the fist in the first 6 ms. Compare this with the position-time graph.

62. The driver of a car going 90.0 km/h suddenly sees the lights of a barrier 40.0 m ahead. It takes the driver 0.75 s to apply the brakes, and the average acceleration during braking is -10.0 m/s^2.
  a. Determine whether the car hits the barrier.
  b. What is the maximum speed at which the car could be moving and not hit the barrier 40.0 m ahead? Assume that the acceleration rate doesn't change.

63. The data in **Table 5–7,** taken from a driver's handbook, show the distance a car travels when it brakes to a halt from a specific initial velocity.
  a. Plot the braking distance versus the initial velocity. Describe the shape of the curve.
  b. Plot the braking distance versus the square of the initial velocity. Describe the shape of the curve.

TABLE 5–7	
Initial Velocity versus Braking Distance	
Initial Velocity (m/s)	**Braking Distance (m)**
11	10
15	20
20	34
25	50
29	70

  c. Calculate the slope of your graph from part **b.** Find the value and units of the quantity 1/slope.
  d. Does this curve agree with the equation $v_0^2 = -2ad$? What is the value of a?

64. As a traffic light turns green, a waiting car starts with a constant acceleration of 6.0 m/s^2. At the instant the car begins to accelerate, a truck with a constant velocity of 21 m/s passes in the next lane.
  a. How far will the car travel before it overtakes the truck?
  b. How fast will the car be traveling when it overtakes the truck?

65. Use the information given in problem 64.
  a. Draw velocity-time and position-time graphs for the car and truck.
  b. Do the graphs confirm the answer you calculated for problem 64?

Section 5.4

66. An astronaut drops a feather from 1.2 m above the surface of the moon. If the acceleration of gravity on the moon is 1.62 m/s^2 downward, how long does it take the feather to hit the moon's surface?

67. A stone falls freely from rest for 8.0 s.
  a. Calculate the stone's velocity after 8.0 s.
  b. What is the stone's displacement during this time?

68. A student drops a penny from the top of a tower and decides that she will establish a coordinate system in which the direction of the penny's motion is positive. What is the sign of the acceleration of the penny?

Chapter 5 Review **113**

69. A bag is dropped from a hovering helicopter. When the bag has fallen 2.0 s,
   a. what is the bag's velocity?
   b. how far has the bag fallen?

70. A weather balloon is floating at a constant height above Earth when it releases a pack of instruments.
   a. If the pack hits the ground with a velocity of −73.5 m/s, how far did the pack fall?
   b. How long did it take for the pack to fall?

71. During a baseball game, a batter hits a high pop-up. If the ball remains in the air for 6.0 s, how high does it rise? **Hint:** Calculate the height using the second half of the trajectory.

72. Table 5–8 gives the positions and velocities of a ball at the end of each second for the first 5.0 s of free fall from rest.
   a. Use the data to plot a velocity-time graph.
   b. Use the data in the table to plot a position-time graph.
   c. Find the slope of the curve at the end of 2.0 s and 4.0 s on the position-time graph. Do the values agree with the table of velocity?
   d. Use the data in the table to plot a position-versus-time-squared graph. What type of curve is obtained?
   e. Find the slope of the line at any point. Explain the significance of the value.
   f. Does this curve agree with the equation $d = 1/2\ gt^2$?

TABLE 5–8		
Position and Velocity in Free Fall		
Time (s)	**Position (m)**	**Velocity (m/s)**
0.0	0.0	0.0
1.0	−4.9	−9.8
2.0	−19.6	−19.6
3.0	−44.1	−29.4
4.0	−78.4	−39.2
5.0	−122.5	−49.0

73. The same helicopter in problem 69 is rising at 5.0 m/s when the bag is dropped. After 2.0 s,
   a. what is the bag's velocity?

   b. how far has the bag fallen?
   c. how far below the helicopter is the bag?

74. The helicopter in problems 69 and 73 now descends at 5.0 m/s as the bag is released. After 2.0 s,
   a. what is the bag's velocity?
   b. how far has the bag fallen?
   c. how far below the helicopter is the bag?

75. What is common to the answers to problems 69, 73, and 74?

76. A tennis ball is dropped from 1.20 m above the ground. It rebounds to a height of 1.00 m.
   a. With what velocity does it hit the ground?
   b. With what velocity does it leave the ground?
   c. If the tennis ball were in contact with the ground for 0.010 s, find its acceleration while touching the ground. Compare the acceleration to *g*.

Extra Practice For more practice solving problems, go to Extra Practice Problems, Appendix B.

Critical Thinking Problems

77. An express train, traveling at 36.0 m/s, is accidentally sidetracked onto a local train track. The express engineer spots a local train exactly 1.00×10^2 m ahead on the same track and traveling in the same direction. The local engineer is unaware of the situation. The express engineer jams on the brakes and slows the express at a constant rate of 3.00 m/s². If the speed of the local train is 11.0 m/s, will the express train be able to stop in time or will there be a collision? To solve this problem, take the position of the express train when it first sights the local train as a point of origin. Next, keeping in mind that the local train has exactly a 1.00×10^2 m lead, calculate how far each train is from the origin at the end of the 12.0 s it would take the express train to stop.

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a. On the basis of your calculations, would you conclude that a collision will occur?

b. The calculations you made do not allow for the possibility that a collision might take place before the end of the 12 s required for the express train to come to a halt. To check this, take the position of the express train when it first sights the local train as the point of origin and calculate the position of each train at the end of each second after sighting. Make a table showing the distance of each train from the origin at the end of each second. Plot these positions on the same graph and draw two lines. Use your graph to check your answer to part **a.**

78. Which has the greater acceleration: a car that increases its speed from 50 to 60 km/h, or a bike that goes from 0 to 10 km/h in the same time? Explain.

79. You plan a car trip on which you want to average 90 km/h. You cover the first half of the distance at an average speed of only 48 km/h. What must your average speed be in the second half of the trip to meet your goal? Is this reasonable? Note that the velocities are based on half the distance, not half the time.

Going Further

Applying Calculators Members of a physics class stood 25 m apart and used stopwatches to measure the time a car driving down the highway passed each person. The data they compiled are shown in **Table 5–9.**

| TABLE 5–9			
Position versus Time			
Time (s)	Position (m)	Time (s)	Position (m)
0.0	0.0	5.9	125.0
1.3	25.0	7.0	150.0
2.7	50.0	8.6	175.0
3.6	75.0	10.3	200.0
5.1	100.0		

Use a graphing calculator to fit a line to a position-time graph of the data and to plot this line. Be sure to set the display range of the graph so that all the data fit on it. Find the slope of the line. What was the speed of the car?

Applying CBLs Design a lab to measure the distance an accelerated object moves over time. Use equal time intervals so that you can plot velocity over time as well as distance. A pulley at the edge of a table with a mass attached is a good way to achieve uniform acceleration. Suggested materials include a motion detector, CBL, lab cart, string, pulley, C-clamp, and mass. Generate graphs of distance versus time and velocity versus time using different masses on the pulley. How did the change in mass affect your graphs?

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