

A Graphic Display

Computers monitor the speed, the location, the fuel consumption, and the environment in and around the spacecraft; the physiological condition of any crew members; and data from experiments. How do scientists use equations and graphs to analyze and display this information?

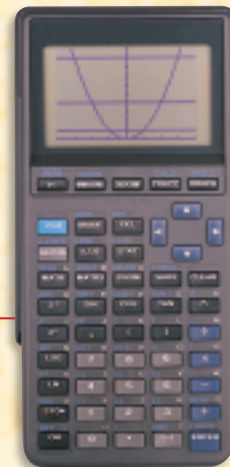
➡ Look at the text on page 33 for the answer.



CHAPTER

2

A Mathematical Toolkit



Within your lifetime, humans may set foot on Mars. What will they find when they arrive? Scientists believe that the physical principles learned on Earth will be equally valid on Mars. After all, it will have been those principles that propelled researchers to Mars in the first place.

How can researchers test this hypothesis? They would make observations, conduct experiments, and take measurements. They would identify what factors or phenomena are related. For example, researchers might find that the force required to move an object and the mass of the object are related. Then they would determine how great a force is needed to move a given mass. Scientists, or perhaps robot space stations, would transmit the results of both the qualitative and quantitative experiments to Earth using words, diagrams, and mathematics.

Methodical or logical thinking lies at the heart of physics. Scientists rely on experiments with numerical results to support their conclusions. The key to the process is the requirement that the experiments, and the results, be reproducible.

Mathematics has often been called the language of physics. In this chapter, you'll find mathematical techniques that will be useful throughout this course. You might think of this chapter as a collection of tools. Appendix A contains additional mathematical tools such as the relationship between decimals, fractions, and percents. There is also a reminder about basic concepts of geometry and trigonometry. Look through Appendix A now to find out what is there. Remember to refer to it as part of your problem-solving approach.

WHAT YOU'LL LEARN

- You will perform calculations using SI units and scientific notation.
- You will understand the need for accuracy and precision when making measurements and reporting data.
- You will display and evaluate data using graphs.

WHY IT'S IMPORTANT

- A basic understanding of mathematics is useful not only in the laboratory but also at the shopping mall, on the highway, in the kitchen, and on the playing field.



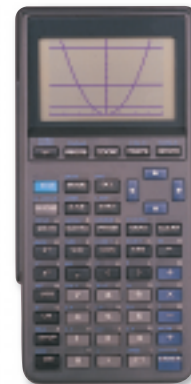
To find out more about the use of mathematics in science, visit the Glencoe Science Web site at science.glencoe.com



CLICK HERE

2.1

The Measures of Science



OBJECTIVES

- **Define** the SI standards of measurement.
- **Use** common metric prefixes.
- **Estimate** measurements and solutions to problems.
- **Perform** arithmetic operations using scientific notation.

The first humans on Mars must make the measurements needed to do experiments on Mars. But, what is a measurement? Every measurement is a comparison between an unknown quantity and a standard. If this comparison is to be valid, the measuring device must be compared against a widely accepted standard. For a standard to be useful, it must be practical for the type of measurement being made, readily accessible, reproducible, and constant over time. There must be agreement among users as to what the standard defines.

The Metric System and SI

French scientists adopted the metric system of measurement in 1795. Until that time, communications among scientists had been difficult because the units of measurement were not standardized. Units of measurement had been based on local customs. The **metric system** provides a set of standards of measurement that is convenient to use because units of different sizes are related by powers of 10.

The worldwide scientific community and most countries currently use an adaptation of the metric system to make measurements. The *Système Internationale d'Unités*, or **SI**, is regulated by the International Bureau of Weights and Measures in Sèvres, France. This bureau and the National Institute of Science and Technology (NIST) in Gaithersburg, Maryland keep the standards of length, time, and mass against which our metersticks, clocks, and balances are calibrated. Because other quantities can be described using combinations of these three units, length, time, and mass are base quantities. The units in which these quantities are measured are thus **base units**. Table 2–1 lists the seven base quantities and their units, which are the foundation of SI.



FIGURE 2–1 The Metric Conversion Act became law in the United States in 1975.

TABLE 2–1

SI Base Units

Base quantity	Base unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Temperature	kelvin	K
Amount of a substance	mole	mol
Electric current	ampere	A
Luminous intensity	candela	cd



The SI base unit of length is the **meter**, m, as shown in **Figure 2-1**. The meter was first defined as 1/10 000 000 of the distance from the north pole to the equator, measured along a line passing through Lyons, France. Later, it was more practical to define the standard meter as the distance between two lines engraved on a platinum-iridium bar kept in Paris. Methods of comparing times have become much more precise than those of comparing lengths. Therefore, in 1983, the meter was defined as the distance traveled by light in a vacuum during a time interval of 1/299 792 458 s.

The standard SI unit of time is the **second**, s. The second was first defined as 1/86 400 of the mean solar day. A mean solar day is the average length of the day over a period of one year, approximately 24 hours. It is now known that Earth's rotation is slowing; and days are getting longer; thus that standard is not constant. In the 1960s, atomic clocks were developed that gain or lose only 1 s in approximately 3 000 000 years. The second is currently defined in terms of the frequency of one type of radiation emitted by a cesium-133 atom. NIST adds a leap second every few years as Earth's rotation continues to slow.

The third standard unit, the **kilogram**, kg, measures the mass of an object. The kilogram is the mass of a small platinum-iridium metal cylinder kept at very controlled temperature and humidity. A copy is kept at NIST, as shown in **Figure 2-2**.

A wide variety of other units, called **derived units**, are combinations of the base units. Common derived units include the meter per second, m/s, used to measure speed, and the joule, $\text{kg} \cdot \text{m}^2/\text{s}^2$, used to measure energy, as shown in **Figure 2-3**. As you learn the base and derived units, you will find it is useful when solving physics problems to perform dimensional analysis, that is, to treat the units in each term of the equation as algebraic quantities, to help assure the accuracy of an answer.



FIGURE 2-2 The International Prototype Kilogram is composed of a platinum-iridium alloy.

Math Handbook



To review **dimensional analysis**, see the Math Handbook, Appendix A, page 740.

PROBLEM SOLVING STRATEGIES

Estimates

Does a measurement you made during an experiment or the answer to a problem you solved make sense? Suppose you calculated that a runner was moving at 275 m/s. Is that number reasonable, or could you have made a mistake? To check your results, you often can make a rough estimate.

The ability to make rough estimates without actually solving a problem is a skill you will find useful all your life. Rough estimates give you a hint about how to start working on a solution and whether or not the method has a chance of working.

For example, is 275 m/s a reasonable running speed? Your stride is about 1 m. How many strides can you make in 1 s? A sprinter runs about 100 m in 10 s. A rough estimate of 10 m/s tells you that your answer of 275 m/s is not reasonable.



FIGURE 2-3 The SI derived unit for energy, the joule, is common on food labels in many countries.

HISTORY CONNECTION

Ancient Measurement

How did people measure in ancient times? In Mesopotamia (3500–1800 B.C.), workers built the first cities using cubits (43–56 cm or 17–22 in). They measured weight in shekels (0.5 oz). In the 1400s and 1500s, the Incas used body parts as units of measurement. A span was the length of a man's hand: 20 cm (8 in). A fathom was the width of a man's outstretched arms: 160 cm (64 in). A pace was 1.2 m (4 ft). What problems would result from using these measurement units?



Rough estimates have to start with reasonable lengths and times. How long is a car? How large is a page in this book? How wide is your classroom? You do not have to look for a ruler or meterstick—you already have three built-in measuring sticks. The distance from your nose to the fingertips on your outstretched hand is about one meter. The width of your fist is about $1/10$ m, and the width of your finger is about $1/100$ m or 1 cm.

How long does it take something to fall? You can count seconds by saying, to yourself, “one chimpanzee, two chimpanzees, and so on” In fact, by placing the emphasis on “**one chimpanzee**,” you can actually estimate half-seconds quite well. You’ll find that it takes about one-half second for an object held high above your head to hit the ground.

With these methods, you can estimate times, distances, and velocities to within a factor of 2 or 3. That is usually good enough to find the worst of your errors.

SI Prefixes

The metric system is a decimal system. Prefixes are used to change SI units by powers of 10. To use SI units effectively, you should know the meanings of the prefixes in **Table 2–2**. For example, $1/10$ of a meter is a decimeter, $1/100$ of a meter is a centimeter, $1/1000$ of a meter is a millimeter and 1000 meters is a kilometer. All of these divisions except the kilometer can be found on a meterstick. **Figure 2–4** shows the vast range of lengths of objects in our universe; commonly used length units also are indicated.

The same prefixes are used for all quantities. For example, $1/1000$ of a gram is a milligram, $1/1000$ of a second is a millisecond, and $1/1000$ of a liter is a milliliter.

TABLE 2–2

Prefixes Used with SI Units				
Prefix	Symbol	Multiplier	Scientific notation	Example
femto	f	1/1 000 000 000 000 000	10^{-15}	femtosecond (fs)
pico	p	1/1 000 000 000 000	10^{-12}	picometer (pm)
nano	n	1/1 000 000 000	10^{-9}	nanometer (nm)
micro	μ	1/1 000 000	10^{-6}	microgram (μ g)
milli	m	1/1000	10^{-3}	milligram (mg)
centi	c	1/100	10^{-2}	centimeter (cm)
deci	d	1/10	10^{-1}	deciliter (dL)
kilo	k	1000	10^3	kilometer (km)
mega	M	1 000 000	10^6	megagram (Mg)
giga	G	1 000 000 000	10^9	gigameter (Gm)
tera	T	1 000 000 000 000	10^{12}	terameter (Tm)



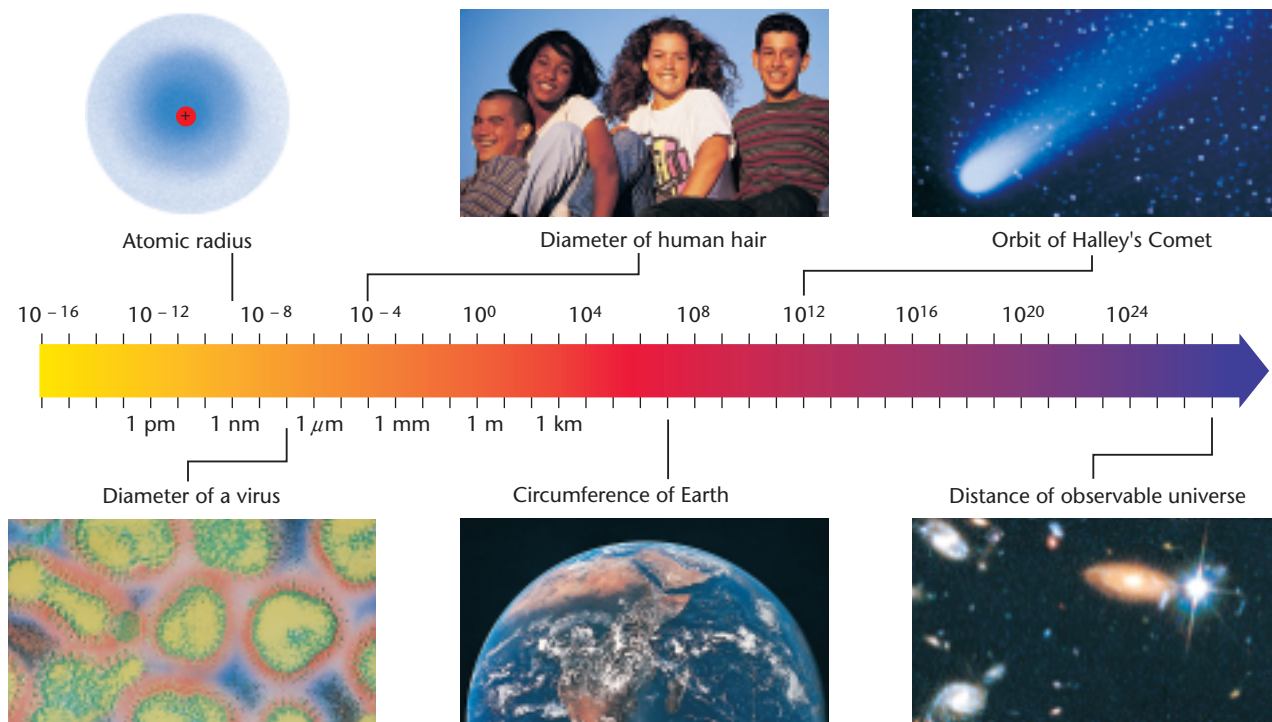


FIGURE 2-4 Objects in the universe range from the very small to the unimaginably large.

Scientific Notation

Many of the numerical values of the multipliers in **Table 2-2** are very large or very small numbers. Written in this form, the values of the quantities take up much space. Such large or small measurements are difficult to read, their relative sizes are difficult to determine, and they are awkward to use in calculations. To work with such numbers, write them in **scientific notation** by expressing decimal places as powers of 10. The numerical part of a quantity is written as a number between 1 and 10 multiplied by a whole-number power of 10.

$$M \times 10^n$$

$1 \leq M < 10$ and n is an integer. To write numbers using scientific notation, move the decimal point until only one non zero digit remains on the left. Then count the number of places you moved the decimal point and use that number as the exponent of 10.

The average distance from the sun to Mars is 227 800 000 000 m. In scientific notation, this distance would be 2.278×10^{11} m. The number of places you move the decimal to the left is expressed as a positive exponent of 10.

The mass of an electron is about

0.000 000 000 000 000 000 000 000 000 911 kg.

To write this number in scientific notation, the decimal point is moved 31 places to the right. As a result the mass of an electron is written as 9.11×10^{-31} kg. The number of places you move the decimal to the right is expressed as a negative exponent of 10.

Pocket Lab

How good is your eye?

The distance from your nose to your outstretched fingertips is about 1 m. Estimate the distance between you and three objects in the room. Have the members in your lab group each make a data table and record their estimates. Verify each distance.

Compare Results Were the estimates reasonably close? Did one person consistently make accurate estimates? What could be done to improve your accuracy?

Scientific notation with calculators Many calculators display numbers in scientific notation as *M En*. For example, a calculator might show 2.278×10^{11} as 2.278 E11 or show 9.11×10^{-31} as 9.11 E - 31. When you report the results of a calculation, you should write it in normal scientific notation.

Practice Problems

- Express the following quantities in scientific notation.
 - 5800 m
 - 450 000 m
 - 302 000 000 m
 - 86 000 000 000 m
- Express the following quantities in scientific notation.
 - 0.000 508 kg
 - 0.000 000 45 kg
 - 0.0003600 kg
 - 0.004 kg
- Express the following quantities in scientific notation.
 - 300 000 s
 - 186 000 s
 - 93 000 000 s

F.Y.I.

Parts used in high-performance car engines must be measured to within $7\mu\text{m}$ (7×10^{-6} meters).

Converting Units

What is the equivalent in kg of 465 g? You know from **Table 2–2** that $1 \text{ kg} = 1000 \text{ g}$. Thus, $(1000 \text{ g})/(1 \text{ kg}) = 1$. How can this information be used to convert units?

PROBLEM SOLVING STRATEGIES

Math Handbook



To review **ratios**, **rates**, and **proportions**, see the Math Handbook, Appendix A, page 739.

The Factor-Label Method of Unit Conversion

An easy way to convert a quantity expressed in one unit to that quantity in another unit is to use a conversion factor, a relationship between the two units. A conversion factor is a multiplier equal to 1. Because $1 \text{ kg} = 1000 \text{ g}$, you can construct the following conversion factors.

$$1 = \frac{1 \text{ kg}}{1000 \text{ g}} \text{ or } 1 = \frac{1000 \text{ g}}{1 \text{ kg}}$$

Recall that the value of a quantity does not change when it is multiplied or divided by 1. Therefore, to find the equivalent in kg of 465 g, multiply it by an appropriate conversion factor.

$$465 \text{ g} = (465 \text{ g})\left(\frac{1 \text{ kg}}{1000 \text{ g}}\right) = \frac{465 \text{ g} \times 1 \text{ kg}}{1000 \text{ g}} = 0.465 \text{ kg}$$

Unit labels cancel just like algebraic quantities. If the final units do not make sense, check your conversion factors. A factor may have been inverted or written incorrectly. This method of converting one unit to another is called the **factor-label method** of unit conversion.

Practice Problems

4. Convert each of the following length measurements as directed.
- a. 1.1 cm to meters c. 2.1 km to meters
b. 76.2 pm to millimeters d. 2.278×10^{11} m to kilometers
5. Convert each of the following mass measurements to its equivalent in kilograms.
- a. 147 g b. 11 Mg c. $7.23 \mu\text{g}$ d. 478 mg

Arithmetic Operations in Scientific Notation

Suppose you need to add or subtract measurements expressed in scientific notation, $M \times 10^n$. The measurements must be expressed in the same powers of 10 and the same units.

Example Problem

Addition and Subtraction Using Scientific Notation

Solve the following problems. Express the answers in scientific notation.

- a. $4 \times 10^8 \text{ m} + 3 \times 10^8 \text{ m}$
b. $4.1 \times 10^{-6} \text{ kg} - 3.0 \times 10^{-7} \text{ kg}$
c. $4.02 \times 10^6 \text{ m} + 1.89 \times 10^2 \text{ m}$

Calculate Your Answer

Strategy:

- a. If the numbers have the same exponent, n , add or subtract the values of M and keep the same n .
- b. If the exponents are not the same, move the decimal to the left or right until they are the same. Then add or subtract M .
- c. If the magnitude of one number is quite small when compared to the other number, its effect on the larger number is insignificant. The smaller number can be treated as zero.

Calculations:

$$\begin{aligned} &4 \times 10^8 \text{ m} + 3 \times 10^8 \text{ m} \\ &= (4 + 3) \times 10^8 \text{ m} \\ &= 7 \times 10^8 \text{ m} \end{aligned}$$

$$\begin{aligned} &4.1 \times 10^{-6} \text{ kg} - 3.0 \times 10^{-7} \text{ kg} \\ &= 4.1 \times 10^{-6} \text{ kg} - 0.30 \times 10^{-6} \text{ kg} \\ &= (4.1 - 0.30) \times 10^{-6} \text{ kg} \\ &= 3.8 \times 10^{-6} \text{ kg} \end{aligned}$$

$$\begin{aligned} &4.02 \times 10^6 \text{ m} + 1.89 \times 10^2 \text{ m} \\ &= 40\,200 \times 10^2 \text{ m} + 1.89 \times 10^2 \text{ m} \\ &= (40\,200 + 1.89) \times 10^2 \text{ m} \\ &= 40\,201.89 \times 10^2 \text{ m} \\ &= 4.020\,189 \times 10^6 \text{ m} \\ &= 4.02 \times 10^6 \text{ m} \end{aligned}$$

Practice Problems

Solve the following problems. Write your answers in scientific notation.

6. a. $5 \times 10^{-7} \text{ kg} + 3 \times 10^{-7} \text{ kg}$
 b. $4 \times 10^{-3} \text{ kg} + 3 \times 10^{-3} \text{ kg}$
 c. $1.66 \times 10^{-19} \text{ kg} + 2.30 \times 10^{-19} \text{ kg}$
 d. $7.2 \times 10^{-12} \text{ kg} - 2.6 \times 10^{-12} \text{ kg}$
7. a. $6 \times 10^{-8} \text{ m}^2 - 4 \times 10^{-8} \text{ m}^2$
 b. $3.8 \times 10^{-12} \text{ m}^2 - 1.90 \times 10^{-11} \text{ m}^2$
 c. $5.8 \times 10^{-9} \text{ m}^2 - 2.8 \times 10^{-9} \text{ m}^2$
 d. $2.26 \times 10^{-18} \text{ m}^2 - 1.8 \times 10^{-18} \text{ m}^2$
8. a. $5.0 \times 10^{-7} \text{ mg} + 4 \times 10^{-8} \text{ mg}$
 b. $6.0 \times 10^{-3} \text{ mg} + 2 \times 10^{-4} \text{ mg}$
 c. $3.0 \times 10^{-2} \text{ pg} - 2 \times 10^{-6} \text{ ng}$
 d. $8.2 \text{ km} - 3 \times 10^2 \text{ m}$

Math Handbook



To review the **properties of exponents**, see the Math Handbook, Appendix A, page 741.

To multiply quantities written in scientific notation, simply multiply the values and units of M . Then add the exponents. To divide quantities expressed in scientific notation, divide the values and units of M , then subtract the exponent of the divisor from the exponent of the dividend. If one unit is a multiple of the other, convert to the same unit.

Example Problem

Multiplication and Division Using Scientific Notation

Find the value of each of the following quantities.

- a. $(4 \times 10^3 \text{ kg})(5 \times 10^{11} \text{ m})$
- b. $\frac{8 \times 10^6 \text{ m}^3}{2 \times 10^{-3} \text{ m}^2}$

Calculate Your Answer

Strategy:

- a. Multiply the values of M and add the exponents, n . Multiply the units.
- b. Divide the values of M and subtract the exponent of the divisor from the exponent of the dividend.

Calculations:

$$\begin{aligned} (4 \times 10^3 \text{ kg})(5 \times 10^{11} \text{ m}) &= (4 \times 5) \times 10^{3+11} \text{ kg} \cdot \text{m} \\ &= 20 \times 10^{14} \text{ kg} \cdot \text{m} \\ &= 2 \times 10^{15} \text{ kg} \cdot \text{m} \end{aligned}$$

$$\begin{aligned} \frac{8 \times 10^6 \text{ m}^3}{2 \times 10^{-3} \text{ m}^2} &= \frac{8}{2} \times 10^{6-(-3)} \text{ m}^{3-2} \\ &= 4 \times 10^9 \text{ m} \end{aligned}$$

Practice Problems

Find the value of each of the following quantities.

9. a. $(2 \times 10^4 \text{ m})(4 \times 10^8 \text{ m})$
 b. $(3 \times 10^4 \text{ m})(2 \times 10^6 \text{ m})$
 c. $(6 \times 10^{-4} \text{ m})(5 \times 10^{-8} \text{ m})$
 d. $(2.5 \times 10^{-7} \text{ m})(2.5 \times 10^{16} \text{ m})$
10. a. $\frac{6 \times 10^8 \text{ kg}}{2 \times 10^4 \text{ m}^3}$ c. $\frac{6 \times 10^{-8} \text{ m}}{2 \times 10^4 \text{ s}}$
 b. $\frac{6 \times 10^8 \text{ kg}}{2 \times 10^{-4} \text{ m}^3}$ d. $\frac{6 \times 10^{-8} \text{ m}}{2 \times 10^{-4} \text{ s}}$
11. a. $\frac{(3 \times 10^4 \text{ kg})(4 \times 10^4 \text{ m})}{6 \times 10^4 \text{ s}}$
 b. $\frac{(2.5 \times 10^6 \text{ kg})(6 \times 10^4 \text{ m})}{5 \times 10^{-2} \text{ s}^2}$
12. a. $(4 \times 10^3 \text{ mg})(5 \times 10^4 \text{ kg})$
 b. $(6.5 \times 10^{-2} \text{ m})(4.0 \times 10^3 \text{ km})$
 c. $(2 \times 10^3 \text{ ms})(5 \times 10^{-2} \text{ ns})$
13. a. $\frac{2.8 \times 10^{-2} \text{ mg}}{2.0 \times 10^4 \text{ g}}$
 b. $\frac{(6 \times 10^2 \text{ kg})(9 \times 10^3 \text{ m})}{(2 \times 10^4 \text{ s})(3 \times 10^6 \text{ ms})}$
14. $\frac{(7 \times 10^{-3} \text{ m}) + (5 \times 10^{-3} \text{ m})}{(9 \times 10^7 \text{ km}) + (3 \times 10^7 \text{ km})}$



USING A CALCULATOR

Scientific Notation

Using a calculator simplifies performing arithmetic operations on numbers in scientific notation.

$$\frac{8 \times 10^6 \text{ kg}}{2 \times 10^{-3} \text{ m}^3}$$

Keys

8 EXP 6 ÷
2 EXP 3 +/- =

Display

8⁰⁶

4⁰⁹

Answer

$$4 \times 10^9 \text{ kg/m}^3$$

$$4.0 \times 10^{-6} \text{ kg} - 3.0 \times 10^{-7} \text{ kg}$$

Keys

4.0 EXP 6 +/- -
3.0 EXP 7 +/- =

Display

4.0⁻⁰⁶

3.7⁻⁰⁶

Answer

$$3.7 \times 10^{-6} \text{ kg}$$

2.1 Section Review

- A calculator displayed a number as 1.574 E8. Express this number in normal scientific notation.
- Your height could be given either in terms of a small unit, such as a millimeter, or a larger unit, such as a meter. In which case would your height be a larger number?
- Describe in detail how you would measure the time in seconds it takes you to go from home to school.
- Critical Thinking** What additional steps would you need to time your trip, using one clock at home and one at school?

Measurement Uncertainties



OBJECTIVES

- **Distinguish** between accuracy and precision.
- **Indicate** the precision of measured quantities with significant digits.
- **Perform** arithmetic operations with significant digits.

Scientists don't believe the result of an experiment or the prediction of a theory because of the fame of the scientist. Rather, they believe it only when other people have repeatedly obtained the same result. A scientific result has to be reproducible. But no scientific result is perfectly exact. Every measurement, whether it is made by a student or a professional scientist, is subject to uncertainty.

Comparing Results

Before exploring the causes of this uncertainty, let's see how results of experiments along with their uncertainties can be compared. Suppose, for example, three students measured the length of a block of wood. Student 1 made repeated measurements, which ranged from 18.5 cm to 19.1 cm. The average of Student 1's measurements was 18.8 cm, as shown in **Figure 2-5**. This result was reported as (18.8 ± 0.3) cm. Student 2 reported finding the block's length to be (19.0 ± 0.2) cm. Student 3 reported a length of (18.3 ± 0.1) cm.

Could you conclude that the three measurements were in agreement? Was Student 1's result reproducible? The results of Students 1 and 2 overlap, that is, they have the lengths 18.8 cm to 19.1 cm in common. However, there is no overlap and, therefore, no agreement, between their results and the result of Student 3.

Accuracy and Precision

Experimental results can be characterized by their precision and their accuracy. How precise and accurate are the measurements of the three students? **Precision** describes the degree of exactness of a measurement. Student 3's measurements were between 18.2 cm and 18.4 cm. That is, they were within ± 0.1 cm. The measurements of the other two students were less precise because they had a larger variation.

The precision depends on the instrument used to make the measurement. Generally the device that has the finest division on its scale produces the most precise measurement. The precision of a measurement is one-half the smallest division of the instrument. For example, the micrometer in **Figure 2-6** has divisions of 0.01 mm. You can measure an object to within 0.005 mm with this device.

The smallest division on a meterstick is a millimeter, as shown in **Figure 2-7**. Thus, with a meterstick you can measure the length of an object to within 0.5 mm. Would you choose the meterstick or the micrometer to make a more precise measurement?

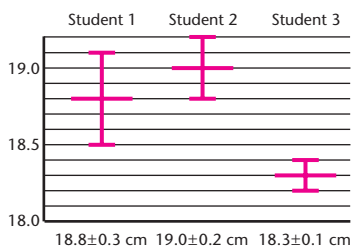


FIGURE 2-5 Three students took multiple measurements of the block of wood. Was Student 1's result reproducible?

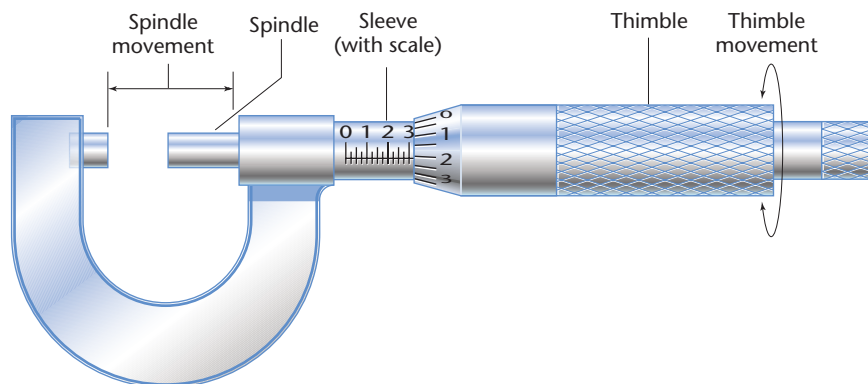


FIGURE 2-6 Micrometers are used to make extremely precise linear measurements.

Accuracy describes how well the results of an experiment agree with the standard value. If the block had been 19.0 cm long, then Student 2 would have been most accurate and Student 3 least accurate.

Although it is possible to make precise measurements with an instrument, those measurements still may not be accurate. The accuracy of the instrument has to be checked. A common method is called the two-point calibration. First, does the instrument read zero when it should? Second, does it give the correct reading when it is measuring an accepted standard? The accuracy of all measuring instruments should be checked regularly.

Techniques of good measurement To assure accuracy and precision, instruments also have to be used correctly. Measurements have to be made carefully if they are to be as precise as the instrument allows. One common source of error comes from the angle at which an instrument is read. Metersticks should either be tipped on their edge or read with the person's eye directly above the stick as shown in **Figure 2-7**.

If the meterstick is read from an angle, the object will appear to be a different length. The difference in the readings is caused by **parallax**, the apparent shift in the position of an object when it is viewed from different angles.

Math Handbook



To review **calculating relative uncertainty** and **relative error** see the Math Handbook, Appendix A, page 737.



FIGURE 2-7 Good technique while taking a measurement will assure accuracy and precision.

HELP WANTED

ACTUARY

Do you love the challenge of figures and probability? A logical, analytical mind and exceptional knowledge of math and statistics are required for the position of actuary in the corporate headquarters of this major insurance company. A bachelor's degree in math or actuarial science and certification on appropriate examinations are absolutely necessary. For more information contact: The American Academy of Actuaries
1100 17th Street, N.W.
7th Floor
Washington, DC 20036

Significant Digits

How long is the metal strip shown in **Figure 2–8**? The smallest division on the meterstick is 0.1 cm. You should read the scale to the nearest 0.1 cm and then estimate any remaining length as a fraction of 0.1 cm. The metal strip in the figure is somewhat longer than 8.6 cm. If by looking closely at the scale, you can see that the end of the strip is about four tenths of the way between 8.6 and 8.7 cm, then, the length of the strip is best stated as 8.64 cm. The last digit is an estimate. It might not be 4, but it is likely not larger than 5 nor less than 3. Your measurement, 8.64 cm, contains three valid digits: the two digits you are sure of, 8 and 6, and one digit, 4, that you estimated. The valid digits in a measurement are called the **significant digits**. If you were to measure the strip with a micrometer, you might find it to be 8.6365 cm long. This measurement would have five significant digits.

Suppose that the end of the strip were exactly on the 8.6 cm mark. In this case, you should record the measurement as 8.60 cm. The zero indicates that the strip is not 0.01 cm more or less than 8.6 cm. The zero is a significant digit because it transmits information. It is the uncertain digit because you are estimating it. The last digit given for any measurement is the uncertain digit. All nonzero digits in a measurement are significant.

Are all zeros significant? Not all zeros are significant. For example, if you had reported the length of the strip in meters as 0.0860 m, it would still have only three significant digits. The first two zeros serve only to locate the decimal point and are not significant. The last zero, however, is the estimated digit and is significant.

How many zeros in the measurement 186 000 m are significant? There is no way to tell. The 6 may be the estimated digit, with the three zeros used to place the decimal point, or they may all have been measured. There are two ways to avoid such confusion. First, the units can be changed to move the decimal point. If the measurement were given as 186 km, it would have three significant digits, but if it were written as 186.000 km, it would have six. Second, it can be written in scientific notation: 1.86×10^5 m has three significant digits and 1.86000×10^5 m has six significant digits.

The following rules summarize how to determine the number of significant digits:

1. Nonzero digits are always significant.
2. All final zeros after the decimal point are significant.
3. Zeros between two other significant digits are always significant.
4. Zeros used solely as placeholders are not significant.

All of the following measurements have three significant digits.

245 m 18.0 g 308 km 0.00623 g

The number of significant digits in a measurement is an indication of the precision with which the measurement was taken.

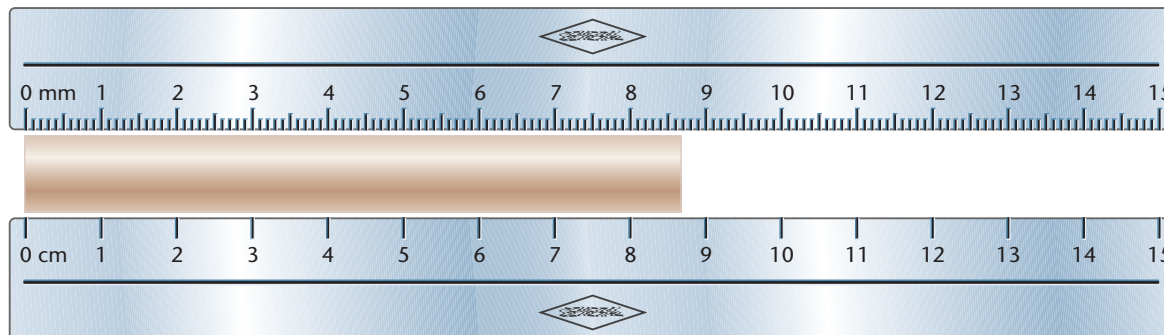


FIGURE 2-8 The accuracy and precision of any measurement depend on both the instrument used and the observer. After a calculation, keep only those digits that truly reflect the accuracy of the original measurement.

Practice Problems

State the number of significant digits in each measurement.

15. a. 2804 m d. 0.003 068 m
 b. 2.84 km e. 4.6×10^5 m
 c. 0.0029 m f. 4.06×10^{-5} m
16. a. 75 m d. 1.87×10^6 mL
 b. 75.00 m e. 1.008×10^8 m
 c. 0.007 060 kg f. 1.20×10^{-4} m

Arithmetic with Significant Digits

When you record the results of an experiment, be sure to record them with the correct number of significant digits. Frequently, you will need to add, subtract, multiply, or divide these measurements. When you perform any arithmetic operation, it is important to remember that the result can never be more precise than the least precise measurement.

To add or subtract measurements, first perform the operation, then round off the result to correspond to the least precise value involved.

Example Problem

Significant Digits: Addition and Subtraction

Add $24.686 \text{ m} + 2.343 \text{ m} + 3.21 \text{ m}$.

Calculate Your Answer

Strategy:

Note that 3.21 m has the least number of decimal places.

Round off the result to the nearest hundredth of a meter.

Calculations:

$$\begin{array}{r} 24.686 \text{ m} \\ 2.343 \text{ m} \\ + 3.21 \text{ m} \\ \hline 30.239 \text{ m} \end{array}$$

The correct answer is 30.24 m.

A different method is used to find the correct number of significant digits when multiplying or dividing measurements. After performing the calculation, note the factor with the least number of significant digits. Round the product or quotient to this number of digits.

Example Problem

Significant Digits: Multiplication and Division

- a. Multiply 3.22 cm by 2.1 cm.
- b. Divide 36.5 m by 3.414 s.

Calculate Your Answer

Strategy:

- a. The factor, 2.1 cm, contains two significant digits. State the product in two significant digits.
- b. The less precise factor contains three significant digits. State the answer in three significant digits.

Calculations:

$$\begin{array}{r} 3.22 \text{ cm} \\ \times 2.1 \text{ cm} \\ \hline 6.762 \text{ cm}^2 \end{array}$$

The correct answer is 6.8 cm².

$$\frac{36.5 \text{ m}}{3.414 \text{ s}} = 10.691 \text{ m/s}$$

The correct answer is 10.7 m/s.

Practice Problems

Solve the following addition problems.

- 17. a. 6.201 cm, 7.4 cm, 0.68 cm, and 12.0 cm
- b. 1.6 km, 1.62 m, and 1200 cm

Solve the following subtraction problems.

- 18. a. 8.264 g from 10.8 g
- b. 0.4168 m from 475 m

Solve the following multiplication problems.

- 19. a. 131 cm \times 2.3 cm
- b. 3.2145 km \times 4.23 km
- c. 5.761 N \times 6.20 m

Solve the following division problems.

- 20. a. 20.2 cm \div 7.41 s
- b. 3.1416 cm \div 12.4 s
- c. 13.78 g \div 11.3 mg
- d. 18.21 g \div 4.4 cm³



FIGURE 2-9 When using a calculator to solve problems, it is important to note that your answers cannot be more precise than the least precise measurement involved.

Some calculators display several additional, meaningless digits, as shown in **Figure 2-9**; some always display only two. Be sure to record your answer with the correct number of digits, as you have just learned in the example problems.

Note that significant digits are only considered when calculating with measurements; there is no uncertainty associated with counting. If, for example, you measure the time required for a race car to make ten counted trips around the track and want to find the average time for one trip, the measured time has an uncertainty, but the number of trips does not.

Significant digits are an important part of interpreting your work and of determining the meaning of your calculations. Be careful about significant digits when you assign them to measurements, when you do arithmetic with those measurements, and when you report the results.

2.2 Section Review

1. You find a micrometer in a cabinet that has been badly bent. How would it compare to a new, high-quality meterstick in precision? In accuracy?
2. Does parallax affect the precision of a measurement that you make? Explain.
3. Your friend tells you that his height is 182 cm. Explain in your own words the range of heights implied by that statement.
4. **Critical Thinking** Your friend states in a report that the time needed for ten laps of a race track had been measured and that the average time required to circle the 2.5-mile track was 65.421 seconds. You know that the clock used had a precision of 0.2 second. How much confidence do you have in the results of the report? Explain.



OBJECTIVES

- **Graph** the relationship between independent and dependent variables.
- **Recognize** linear and direct relationships and **interpret** the slope of the curve.
- **Recognize** quadratic and inverse relationships.

A well-designed graph is like a “picture worth a thousand words.” It often can give you more information than words, columns of numbers, or equations alone. To be useful, however, a graph must be drawn properly. In this section, you will develop graphing techniques that will enable you to display data.

Graphing Data

One of the most important skills to learn in driving a car is how to stop it safely. No car can actually “stop on a dime.” The faster a car is going, the farther it travels before it stops. If you studied to earn a driver’s license, you probably found a table in the manual showing how far a car moves beyond the point at which the driver makes a decision to stop.

Many driving manuals also show the distance that a car travels between the time the driver decides to stop the car and the time the driver puts on the brakes. This is called the reaction distance. When the brakes are applied, the car slows down and travels the braking distance. These distances are shown in **Table 2–3** and the bar graph in **Figure 2–10a**. The total stopping distance for various speeds is the sum of the reaction distance and the braking distance. **Table 2–3** shows the English units used in driver’s manuals and their SI equivalents.

TABLE 2–3							
Reaction and Braking Distances Versus Speed							
Original Speed		Reaction Distance		Braking Distance		Total Distance	
m/s	mph	m	ft	m	ft	m	ft
11	25	8	27	10	34	18	61
16	35	12	38	20	67	32	105
20	45	15	49	34	110	49	159
25	55	18	60	50	165	68	225
29	65	22	71	70	231	92	302

The first step in analyzing data is to look at them carefully. Which variable does the experimenter (the driver) change? In this example, it is the speed of the car. Thus, speed is the independent variable, the variable that is changed or manipulated. The independent variable is the one the experimenter can control directly. The other two variables, reaction distance and braking distance, change as a result of the change in speed. These quantities are called dependent variables, or responding variables. The value of the dependent variable depends on the independent variable.

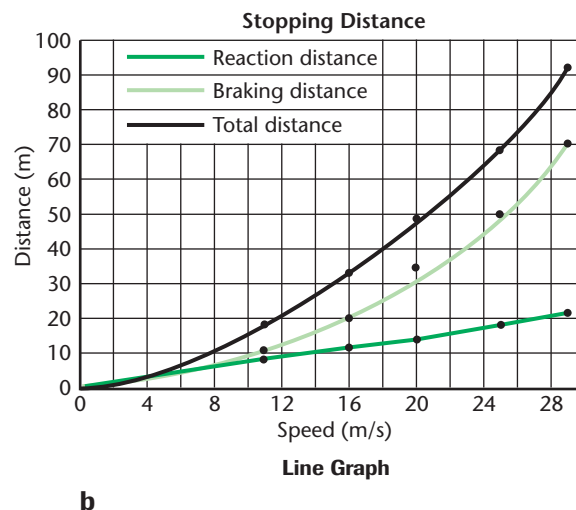
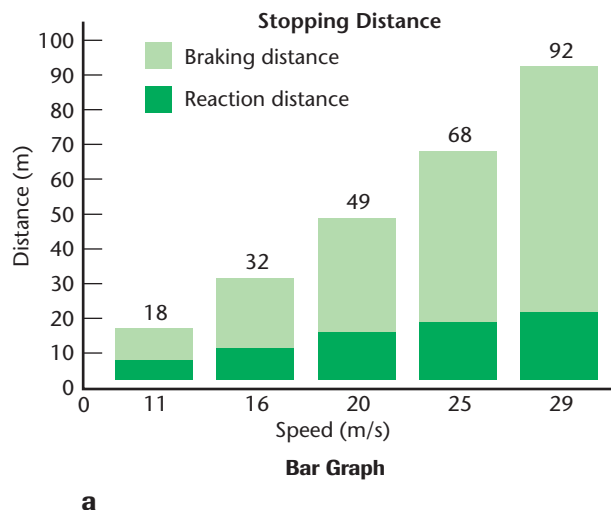


FIGURE 2-10 Graphs (a) and (b) display the same information in two different ways.

How do the distances change for a given change in the speed of the car? Notice that the reaction distance increases 3 to 4 meters for each 5-m/s increase in speed. The braking distance, however, increases by 10 m when the speed increases from 11 to 16 m/s, and by 20 m when the speed increases from 25 to 29 m/s. The way the two distances depend on speed can be seen more easily when the data are plotted on the line graph in **Figure 2-10b**. Follow the steps in the Problem Solving Strategy to create line graphs that display data from tables.

PROBLEM SOLVING STRATEGIES

Plotting Line Graphs

Use the following steps to plot line graphs from data tables.

1. Identify the independent and dependent variables in your data. The independent variable is plotted on the horizontal axis, or x -axis. The dependent variable is plotted on the vertical axis, or y -axis.
2. Determine the range of the independent variable to be plotted.
3. Decide whether the origin (0,0) is a valid data point.
4. Spread the data out as much as possible. Let each division on the graph paper stand for a convenient unit.
5. Number and label the horizontal axis.
6. Repeat steps 2–5 for the dependent variable.
7. Plot the data points on the graph.
8. Draw the “best fit” straight line or smooth curve that passes through as many data points as possible. Do not use a series of straight line segments that “connect the dots.”
9. Give the graph a title that clearly tells what the graph represents.

Pocket Lab

How far around?



Use a meterstick to measure the diameter of four circular objects and a string to measure their circumferences. Record your data in a table. Graph the circumference versus the diameter.

Communicate Results Write a few sentences to summarize your graph. Write a sentence using the word that explains the meaning of the slope of your graph. Explain whether the value of the slope would be different if you had measured in different units.

Mystery Plot

Problem

Can you accurately predict the unknown mass of an object by making measurements of other similar objects?

Hypothesis

Form a hypothesis that relates the mass of an object to another measurable quantity. Describe the variables to be measured and why these measurements are necessary.

Possible Materials



- 4 pieces of electrical wire with lengths between 5 cm and 30 cm
- 3 rectangular pieces of floor tile
- 1 triangular piece of floor tile
- metric ruler
- balance
- graph paper

Plan the Experiment

1. As a group, examine the pieces of floor tile and the pieces of electrical wire. Determine the quantities you want to measure. How can you assure the accuracy and precision of your measurements?
2. Identify the independent and dependent variables.
3. Which objects will be the unknown objects? Which objects will be measured? Set aside the unknowns.
4. Construct a data table or spreadsheet that will include all your measurements and calculations.
5. **Check the Plan** Make sure your teacher has approved your final plan before you proceed with your experiment.
6. Recycle and put away materials that can be reused when you are finished.



Analyze and Conclude

1. **Graphing Data** Make graphs of your measurements to observe relationships between variables. Clearly label the axes.
2. **Analyzing Graphs** Identify the relationship between variables. Do your graphs depict linear, quadratic, or inverse relationships? How do you know? Can you calculate the slope of each graph? Organize, analyze, evaluate, and make inferences in trends from your data. Predict from the trends in your data whether or not your graphs will go through the origin (0,0). Should they?
3. **Calculating Results** Write the equations that relate your variables. Use the equations and the graphs to predict the unknown mass of wire and floor tile.
4. **Checking Your Hypothesis** Measure the unknown masses of the wire and floor tile on the balance. Do your measurements agree with the predicted values?
5. **Calculating Results** Use a computer plotting program or a graphing calculator to re-plot your data and find the equations that relate your variables. Are the equations the same as you found earlier?

Apply

1. Suppose another group measures longer wires. How should the slope of your graph compare to their slope?
2. In the pharmaceutical industry, how might the weight of compressed medicine tablets be used to determine the quantity of finished tablets produced in a specific lot?

In **Figure 2–10b**, speed is plotted on the x -axis, and distance is plotted on the y -axis. Data are given for speeds between 11 and 29 m/s, so a convenient range for the x -axis is 0 to 30 m/s. On the y -axis, the maximum distance is 92 m, so a range of 0 to 100 m is used. When the speed is zero, reaction and stopping distances are both zero, so the graph includes the origin. One division equals 2 m/s on the x -axis and 10 m on the y -axis.

Linear Relationships

Look at **Figure 2–11**, the graph of reaction distance versus speed. A straight line can be drawn through all data points. That is, the dependent variable varies linearly with the independent variable; the two variables are directly proportional. There is a **linear relationship** between the two variables.

The relationship between the two variables can be written as an equation.

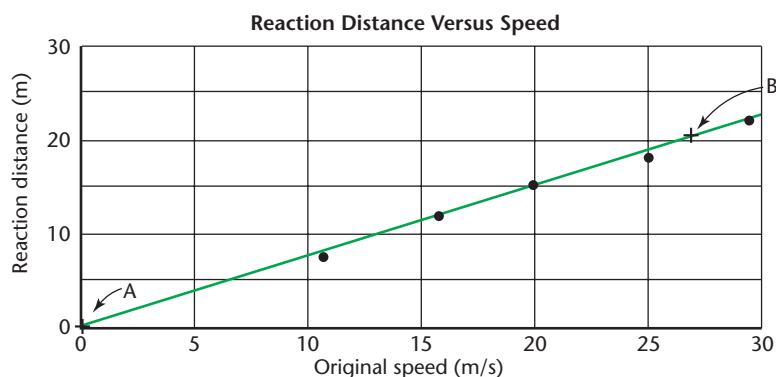
Linear Relationship Between Two Variables $y = mx + b$

The **slope**, m , is the ratio of the vertical change to the horizontal change. To find the slope, select two points, A and B, as far apart as possible on the line. These should not be data points, but points on the line. The vertical change, or rise, Δy , is the difference between the vertical values of A and B. The horizontal change, or run, Δx , is the difference between the horizontal values of A and B. The slope of the graph is then calculated in the following way using points (0,0) and (27,21) from **Figure 2–11**. Note that the units are kept with the variables.

$$\text{Slope } m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$$

$$m = \frac{(21 - 0) \text{ m}}{(27 - 0) \text{ m/s}} = \frac{21 \text{ m}}{27 \text{ m/s}} = 0.78 \text{ s}$$

The **y -intercept**, b , is the point at which the line crosses the y -axis, and it is the y value when the value of x is zero. When the y -intercept is zero, that is, $b = 0$, the equation becomes $y = mx$. The quantity y varies directly with x . The value of y does not always increase with increasing x . If y gets smaller as x gets larger, then $\Delta y/\Delta x$ is negative.



A Graphic Display

➔ *Answers question from page 14.*



FIGURE 2–11 The graph indicates a linear relationship between reaction distance and speed.

Math Handbook



To review the **quadratic formula** and **solving equations**, see the Math Handbook, Appendix A, pages 740, 742.

Nonlinear Relationships

Figure 2–12a is a graph of braking distance versus speed. Note that the graph is not a straight line; the relationship is not linear. The smooth line drawn through all the data points curves upward. Sometimes, such a graph is a parabola, in which the two variables are related by a **quadratic relationship**, represented by the following equation.

$$\text{Quadratic Relationship Between Two Variables } y = ax^2 + bx + c$$

The parabolic relationship exists when one variable depends on the square of another.

A computer program or graphing calculator can easily find the values of the constants a , b , and c in this equation. In later chapters you will learn about variables that are related by this equation and learn why braking distance depends on speed in this way.

Some variables are related by the type of graph that is shown in **Figure 2–12b**. In this case, a plot has been made of the time required to travel a fixed distance as the speed is changed. When the speed is doubled, the time is reduced to one-half its original value. The relationship between speed and time is an **inverse relationship**. The graph is a hyperbola described by the general equation

$$\text{Inverse Relationship } y = \frac{a}{x} \text{ or } xy = a$$

where a is a constant. A hyperbola results when one variable depends on the inverse of the other.

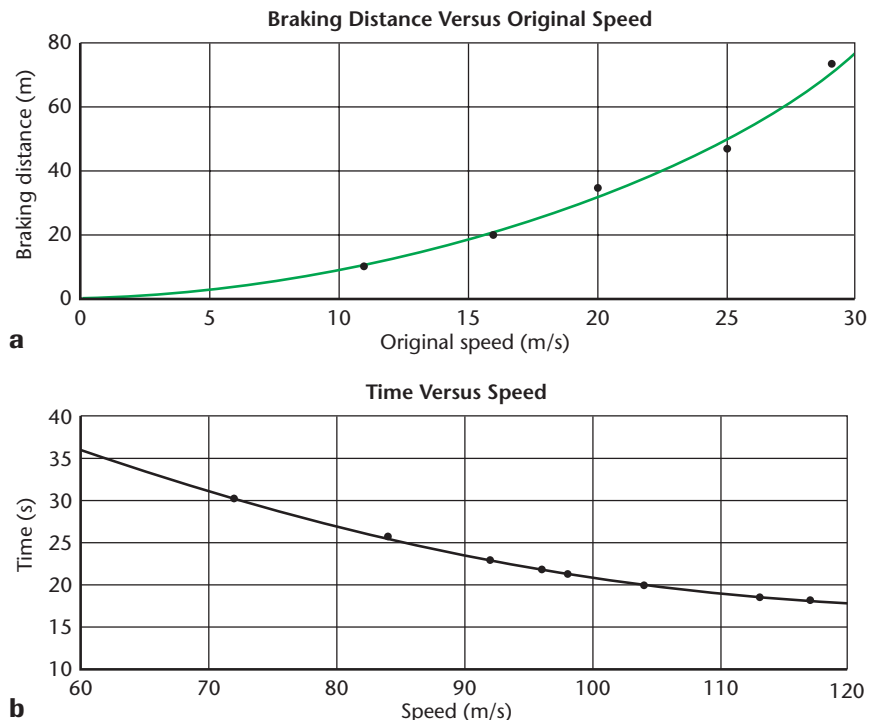


FIGURE 2–12 The graph (a) indicates a parabolic relationship; braking distance varies as the square of the original speed. The graph (b) shows the inverse relationship between the time required to travel a fixed distance and the speed.

How It Works

Electronic Calculators

A pocket calculator is a specialized computer programmed to solve arithmetic problems. The parts of a typical calculator include a power supply, a keypad for entering numbers and calculation commands, and a screen for displaying input numbers and calculation results. The brain of the calculator is a tiny silicon semiconductor chip. This chip, the calculator's processing unit, performs arithmetic operations.

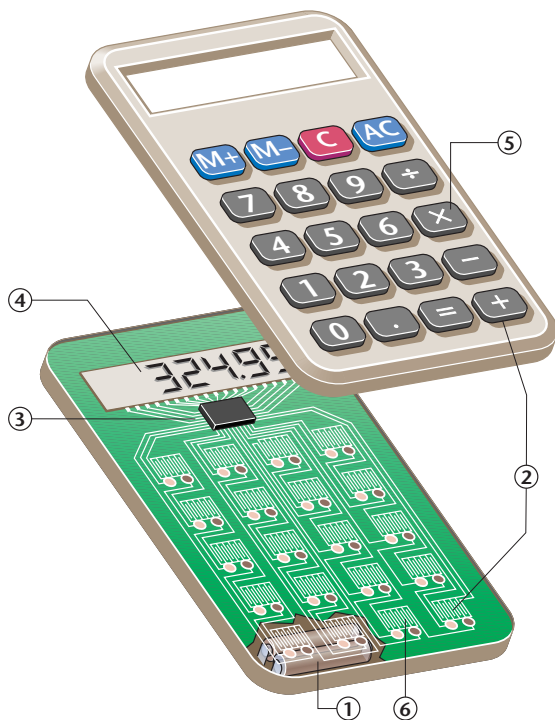


1 Batteries or solar cells provide electricity.

2 Pressing a number on the keypad closes a contact between the key and the circuit board beneath it. The closed contact allows an electrical signal specific to that key to flow from the circuit board to a storage area in the calculator's processing unit.

3 The processing unit's storage area, or memory, holds all input information until the entire problem has been entered and is ready for processing.

4 With each key stroke, an electrical signal also flows from the processing unit to the screen, which displays the number.



5 When one of the function keys such as plus, minus, addition, subtraction, or square root is pressed, its unique signal is also sent to the processing unit for storage. In most calculators, this information is not displayed on the screen.

6 Pressing the equal sign sends a signal to the processing unit instructing it to perform the calculation stored in its memory. The result is sent to the screen for display.

Thinking Critically

1. What are some of the similarities and differences between pocket calculators and computers?
2. Why is it necessary to clear the memory of a calculator before beginning a new problem?

Practice Problems

21. The total distance a lab cart travels during specified lengths of time is given in the following data table.



TABLE 2-4

Time (s)	Distance (m)
0.0	0.00
1.0	0.32
2.0	0.60
3.0	0.95
4.0	1.18
5.0	1.45

- Plot distance versus time from the values given in the table and draw the curve that best fits all points.
- Describe the resulting curve.
- According to the graph, what type of relationship exists between the total distance traveled by the lab cart and the time?
- What is the slope of this graph?
- Write an equation relating distance and time for this data.

2.3 Section Review

- What would be the meaning of a nonzero y -intercept to a graph of reaction distance versus speed?
- Use the graph in **Figure 2-11** to determine at what speed a car moves 10 m while the driver is reacting.
- Explain in your own words the meaning of a steeper line, or greater slope, to the graph of reaction distance versus speed.
- Critical Thinking** The relationship between the circumference and the diameter of a circle is shown in **Figure 2-13**. Could a different

straight line describe a different circle?
What is the meaning of the slope?

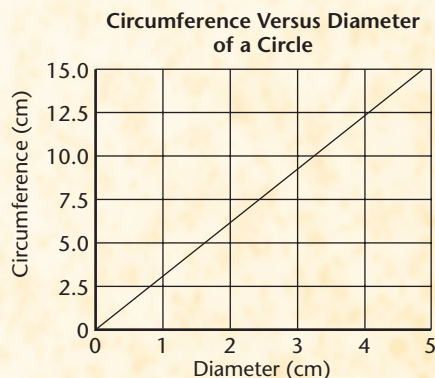


FIGURE 2-13

CHAPTER 2 REVIEW

Summary

Key Terms

2.1

- metric system
- SI
- base unit
- meter
- second
- kilogram
- derived unit
- scientific notation
- factor-label method

2.2

- precision
- accuracy
- parallax
- significant digits

2.3

- linear relationship
- slope
- y-intercept
- quadratic relationship
- inverse relationship

2.1 The Measures of Science

- The meter, second, and kilogram are the SI base units of length, time, and mass, respectively.
- Derived units are combinations of base units.
- Making rough estimates is a good way to start and to check the solution of a problem.
- Prefixes are used to change SI units by powers of 10.
- Very large and very small measurements are most clearly written using scientific notation.
- The method of converting one unit to another unit is called the factor-label method of unit conversion.
- To be added or subtracted, measurements written in scientific notation must be raised to the same power of 10.
- Measurements written in scientific notation need not have the same power of 10 to be multiplied or divided.

2.2 Measurement Uncertainties

- All measurements are subject to some uncertainty.
- Precision is the degree of exactness with which a quantity is measured using a given instrument.
- Accuracy is the extent to which the measured and accepted values of a quantity agree.
- The number of significant digits is limited by the precision of the measuring device.

- The last digit in a measurement is always an estimate.
- The result of any mathematical operation with measurements can never be more precise than the least precise measurement involved in the operation.

2.3 Visualizing Data

- Data are plotted in graphical form to show the relationship between two variables.
- The independent variable is the variable that the experimenter changes. It is plotted on the x - or horizontal axis.
- The dependent variable, which changes as a result of the changes made to the independent variable, is plotted on the y - or vertical axis.
- A graph in which data points lie in a straight line is a graph of a linear relationship.
- A linear relationship can be represented by the equation $y = mx + b$.
- The slope, m , of a straight-line graph is the vertical change (rise) divided by the horizontal change (run).
- The graph of a quadratic relationship is a parabolic curve. It is represented by the equation $y = ax^2 + bx + c$.
- The graph of an inverse relationship between x and y is a hyperbolic curve. It is represented by the equation $y = \frac{a}{x}$.



Key Equations

2.3

$$y = mx + b$$

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x}$$

$$y = ax^2 + bx + c$$

$$xy = a$$

Reviewing Concepts

Section 2.1

1. Why is SI important?
2. List the common SI base units.
3. How are base units and derived units related?
4. You convert the speed limit of an expressway given in miles per hour into meters per second and obtain the value 1.5 m/s. Is this calculation likely to be correct? Explain.
5. Give the name for each multiple of the meter.
 - a. 1/100 m
 - b. 1/1000 m
 - c. 1000 m
6. How many units be used to check on whether a conversion factor has been used correctly?

Section 2.2

7. What determines the precision of a measurement?
8. Explain how a measurement can be precise but not accurate.
9. How does the last digit differ from the other digits in a measurement?
10. Your lab partner recorded a measurement as 100 g.
 - a. Why is it difficult to tell the number of significant digits in this measurement?
 - b. How can the number of significant digits in such a number be made clear?

Section 2.3

11. How do you find the slope of a linear graph?
12. A person who has recently consumed alcohol usually has longer reaction times than a person who has not. Thus, the time between seeing a stoplight and hitting the brakes would be longer for the drinker than for the nondrinker.
 - a. For a fixed speed, would the reaction distance for a driver who had consumed alcohol be longer or shorter than for a nondrinking driver?
 - b. Would the slope of the graph of that reaction distance versus speed have the steeper or the more gradual slope?
13. During a laboratory experiment, the temperature of the gas in a balloon is varied and the volume of the balloon is measured. Which quantity is the independent variable? Which quantity is the dependent variable?

14. For a graph of the experiment in problem 13,
 - a. What quantity is plotted on the horizontal axis?
 - b. What quantity is plotted on the vertical axis?
15. A relationship between the independent variable x and the dependent variable y can be written using the equation $y = ax^2$, where a is a constant.
 - a. What is the shape of the graph of this equation?
 - b. If you define a quantity $z = x^2$, what would be the shape of the graph obtained by plotting y versus z ?
16. Given the equation $F = mv^2/R$, what relationship exists between
 - a. F and R ?
 - b. F and m ?
 - c. F and v ?
17. Based on the equation in problem 16, what type of graph would be drawn for
 - a. F versus R ?
 - b. F versus m ?
 - c. F versus v ?

Applying Concepts

18. The density of a substance is its mass per unit volume.
 - a. Give a possible metric unit for density.
 - b. Is the unit for density base or derived?
19. Use **Figure 2-4** to locate the size of the following objects.
 - a. The width of your thumb.
 - b. The thickness of a page in this book.
 - c. The height of your classroom.
 - d. The distance from your home to your classroom.
20. Make a chart of sizes of objects similar to the one shown in **Figure 2-4**. Include only objects that you have measured. Some should be less than one millimeter; others should be several kilometers.
21. Make a chart similar to **Figure 2-4** of time intervals. Include intervals like the time between heartbeats, the time between presidential elections, the average lifetime of a human, the age of the United States. Find as many very short and very long examples as you can.

22. Three students use a meterstick to measure the width of a lab table. One records a measurement of 84 cm, another of 83.8 cm, and the third of 83.78 cm. Explain which answer is recorded correctly.
23. Two students measure the speed of light. One obtains $(3.001 \pm 0.001) \times 10^8$ m/s; the other obtains $(2.999 \pm 0.006) \times 10^8$ m/s.
 - a. Which is more precise?
 - b. Which is more accurate?
24. Why can quantities with different units never be added or subtracted but can be multiplied or divided? Give examples to support your answer.
25. Suppose you receive \$5.00 at the beginning of a week and spend \$1.00 each day for lunch. You prepare a graph of the amount you have left at the end of each day for one week. Would the slope of this graph be positive, zero, or negative? Why?
26. Data are plotted on a graph and the value on the y -axis is the same for each value of the independent variable. What is the slope? Why?
27. The graph of braking distance versus car speed is part of a parabola. Thus, we write the equation $d = av^2 + bv + c$. The distance, d , has units meters, and velocity, v , has units meter/second. How could you find the units of a , b , and c ? What would they be?
28. In baseball, there is a relationship between the distance the ball is hit and the speed of the pitch. The speed of the pitch is the independent variable. Choose your own relationship. Determine which is the independent variable and which is the dependent variable. If you can, think of other possible independent variables for the same dependent variables.
29. Aristotle said that the quickness of a falling object varies inversely with the density of the medium through which it falls.
 - a. According to Aristotle, would a rock fall faster in water (density 1000 kg/m^3), or in air (density 1 kg/m^3)?
 - b. How fast would a rock fall in a vacuum? Based on this, why would Aristotle say that there could be no such thing as a vacuum?

Problems

Section 2.1

30. Express the following numbers in scientific notation:
 - a. 5 000 000 000 000 m
 - b. 0.000 000 000 166 m
 - c. 2 003 000 000 m
 - d. 0.000 000 103 0 m
31. Convert each of the following measurements to meters.
 - a. 42.3 cm d. 0.023 mm
 - b. 6.2 pm e. 214 μm
 - c. 21 km f. 570 nm
32. Add or subtract as indicated.
 - a. $5.80 \times 10^9 \text{ s} + 3.20 \times 10^8 \text{ s}$
 - b. $4.87 \times 10^{-6} \text{ m} - 1.93 \times 10^{-6} \text{ m}$
 - c. $3.14 \times 10^{-5} \text{ kg} + 9.36 \times 10^{-5} \text{ kg}$
 - d. $8.12 \times 10^7 \text{ g} - 6.20 \times 10^6 \text{ g}$
33. Rank the following mass measurements from smallest to largest: 11.6 mg, 1021 μg , 0.000 006 kg, 0.31 mg.

Section 2.2

34. State the number of significant digits in each of the following measurements.
 - a. 0.000 03 m c. 80.001 m
 - b. 64.01 fm d. 0.720 μg
35. State the number of significant digits in each of the following measurements.
 - a. $2.40 \times 10^6 \text{ kg}$
 - b. $6 \times 10^8 \text{ kg}$
 - c. $4.07 \times 10^{16} \text{ m}$
36. Add or subtract as indicated.
 - a. $16.2 \text{ m} + 5.008 \text{ m} + 13.48 \text{ m}$
 - b. $5.006 \text{ m} + 12.0077 \text{ m} + 8.0084 \text{ m}$
 - c. $78.05 \text{ cm}^2 - 32.046 \text{ cm}^2$
 - d. $15.07 \text{ kg} - 12.0 \text{ kg}$
37. Multiply or divide as indicated.
 - a. $(6.2 \times 10^{18} \text{ m})(4.7 \times 10^{-10} \text{ m})$
 - b. $(5.6 \times 10^{-7} \text{ m})/(2.8 \times 10^{-12} \text{ s})$
 - c. $(8.1 \times 10^{-4} \text{ km})(1.6 \times 10^{-3} \text{ km})$
 - d. $(6.5 \times 10^5 \text{ kg})/(3.4 \times 10^3 \text{ m}^3)$
38. Using a calculator, Chris obtained the following results. Give the answer to each operation using the correct number of significant digits.
 - a. $5.32 \text{ mm} + 2.1 \text{ mm} = 7.4200000 \text{ mm}$
 - b. $13.597 \text{ m} \times 3.65 \text{ m} = 49.62905 \text{ m}^2$
 - c. $83.2 \text{ kg} - 12.804 \text{ kg} = 70.3960000 \text{ kg}$

39. A rectangular floor has a length of 15.72 m and a width of 4.40 m. Calculate the area of the floor.
40. A water tank has a mass of 3.64 kg when it is empty and a mass of 51.8 kg when it is filled to a certain level. What is the mass of the water in the tank?
41. A lawn is 33.21 m long and 17.6 m wide.
- a. What length of fence must be purchased to enclose the entire lawn?
 - b. What area must be covered if the lawn is to be fertilized?
42. The length of a room is 16.40 m, its width is 4.5 m, and its height is 3.26 m. What volume does the room enclose?
43. The sides of a quadrangular plot of land are 132.68 m, 48.3 m, 132.736 m, and 48.37 m. What is the perimeter of the plot?

Section 2.3

44. Figure 2–14 shows the mass of three substances for volumes between 0 and 60 cm³.
- a. What is the mass of 30 cm³ of each substance?
 - b. If you had 100 g of each substance, what would their volumes be?
 - c. In one or two sentences, describe the meaning of the steepness of the lines in this graph.

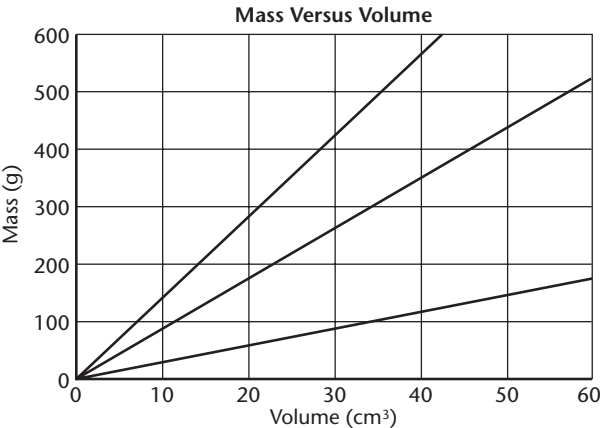


FIGURE 2–14

45. During an experiment, a student measured the mass of 10.0 cm³ of alcohol. The student then measured the mass of 20.0 cm³ of alcohol. In this way, the data in Table 2–5 were collected.

TABLE 2–5	
Volume (cm ³)	Mass (g)
10.0	7.9
20.0	15.8
30.0	23.7
40.0	31.6
50.0	39.6

- a. Plot the values given in the table and draw the curve that best fits all points.
 - b. Describe the resulting curve.
 - c. Use the graph to write an equation relating the volume to the mass of alcohol.
 - d. Find the units of the slope of the graph. What is the name given to this quantity?
46. During a class demonstration, a physics instructor placed a 1.0-kg mass on a horizontal table that was nearly frictionless. The instructor then applied various horizontal forces to the mass and measured the rate at which it gained speed (was accelerated) for each force applied. The results of the experiment are shown in Table 2–6.

TABLE 2–6	
Force (N)	Acceleration (m/s ²)
5.0	4.9
10.0	9.8
15.0	15.2
20.0	20.1
25.0	25.0
30.0	29.9

- a. Plot the values given in the table and draw the curve that best fits the results.
 - b. Describe, in words, the relationship between force and acceleration according to the graph.
 - c. Write the equation relating the force and the acceleration that results from the graph.
 - d. Find the units of the slope of the graph.
47. The physics instructor who performed the experiment in problem 46 changed the procedure. The mass was varied while the force was kept constant. The acceleration of each mass was recorded. The results of the experiment are shown in **Table 2–7**.



TABLE 2–7	
Mass (kg)	Acceleration (m/s ²)
1.0	12.0
2.0	5.9
3.0	4.1
4.0	3.0
5.0	2.5
6.0	2.0

- a. Plot the values given in the table and draw the curve that best fits all points.
- b. Describe the resulting curve.
- c. According to the graph, what is the relationship between mass and the acceleration produced by a constant force?
- d. Write the equation relating acceleration to mass given by the data in the graph.
- e. Find the units of the constant in the equation.



Extra Practice For more practice solving problems, go to **Extra Practice Problems, Appendix B**.

Critical Thinking Problems

48. Find the approximate time needed for a pitched baseball to reach home plate. Report your result to one significant digit. (Use a reference source to find the distance thrown and the speed of a fastball.)

49. Have a student walk across the front of the classroom. Estimate his or her walking speed.
50. How high can you throw a ball? Find a tall building whose height you can estimate and compare the height of your throw to that of the building.
51. Use a graphing calculator or computer graphing program to graph reaction and braking distances versus original speed. Use the calculator or computer to find the slope of the reaction distance and the best quadratic fit to the braking distance.
52. If the sun suddenly ceased to shine, how long would it take Earth to become dark? You will have to look up the speed of light in a vacuum and the distance from the sun to Earth. How long would it take to become dark on the surface of Jupiter?

Going Further

Team Project Divide your class into teams. Estimate the number of students taking high school physics in the United States. Several approaches are possible. You could use the number of students taking physics in your school, the number of students in your class, and the population of your town and scale them to the population of the country. Another approach would be to estimate the proportion of the total number of students in a single grade to the population of the country; then assume that the percentage of physics students in your school holds true for all schools in which physics is offered. Compare the teams' estimates. Assume that each estimate is reported as $\pm 1\%$. Can you conclude that the estimates are in agreement? How could you determine the accuracy of your estimates?

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